INCLUSION IN TEXAS

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 19

Multiplication and Division of Integers

Module 19: Multiplication and Division of Integers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
absolute value	The distance of a number from 0 on a number line.
divide/division	To separate into equal groups.
dividend	The number that is to be divided in a division problem.
divisor	The number that the dividend is divided by.
factor	A number that you multiply with another number to get the product.
integer	A positive or negative whole number.
multiply/multiplication	The process of adding a number to itself a number of times.
negative number	Any number less than 0.
number line	A straight line with numbers placed at equal intervals along its length.
opposites	Two numbers that are equal distance from 0 on a number line.
positive number	Any number greater than 0.
product	The result of multiplying two or more factors.
quotient	The number that results when one number is divided by another number.
zero pair	A pair of numbers with a sum of 0.

B. Background Information

In this module, we focus on multiplication and division of integers. An integer is a positive or negative whole number. We use the following different models to help students understand multiplication and division of integers:

(1) Multiplication with a Number Line
(2) Division with a Number Line
(3) Multiplication with a Quadrant Mat and Cubes
(4) Division with a Quadrant Mat and Cubes

When referring to integers, be sure to emphasize that numbers without a negative symbol (-) are assumed positive. So:

7 is “positive seven” or “seven.”
-7 is “negative seven.”

C. Routines and Examples

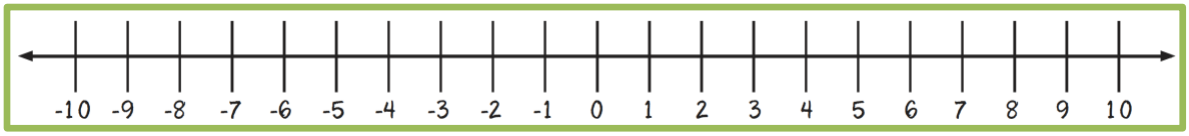
(1) Multiplication with a Number Line

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



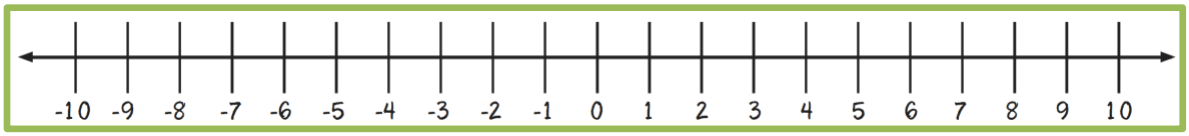
Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we multiplying?**
Students	__ times __.
Teacher	**So, let's start with the first factor. What's the first factor?**
Students	__.
Teacher	**Let's place the duck on the number line at zero. Where do we place the duck?**
Students	At zero.

Teacher	**If the first factor is positive, the duck will face the increasing numbers on the number line. When does the duck face the increasing numbers?**
Students	When the first factor is positive.
Teacher	**If the first factor is negative, the duck will face the decreasing numbers on the number line. When does the duck face the decreasing numbers?**
Students	When the first factor is negative.
Teacher	**So, which way will the duck face in this problem?**
Students	Increasing/decreasing.
Teacher	**Yes, the first factor is positive/negative, so the duck faces the increasing/decreasing numbers.**
Students	(Place duck on zero. Make sure duck is facing increasing/decreasing numbers on the number line.)
Teacher	**Now, let's multiply. What is the second factor?**
Students	__.
Teacher	**If the second factor is positive, the duck will move forward from its position. When does the duck move forward?**
Students	When the second factor is positive.
Teacher	**If the second factor is negative, the duck will move backward from its position. When does the duck move backward?**
Students	When the second factor is negative.
Teacher	**So, which direction should we move?**
Students	Forward/backward.
Teacher	**Because the second factor is positive/negative, we move forward/backward. The second factor is __ so we'll move by jumps of __** (second factor)**. Let's do that together. Count with me.**
Students	__, __, __, …
Teacher	**So, our duck shows the product. What's __ times __?**
Students	__.
Teacher	**Yes. __ times __ equals __. Using this number line helps you understand what it means to multiply integers. How can you use the number line to multiply integers?**
Students	Start at zero. The duck faces increasing numbers with a positive factor and decreasing numbers with a negative factor. Then, the duck jumps the second factor by moving forward if it's a positive factor or backward if it's a negative factor.

Example

-3 × (-2)

EXAMPLE WITH NUMBER LINE



Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we multiplying?**
Students	-3 times -2.
Teacher	**So, let's start with the first factor. What's the first factor?**
Students	-3.
Teacher	**Let's place the duck on the number line at zero. Where do we place the duck?**
Students	At zero.
Teacher	**If the first factor is positive, the duck will face the increasing numbers on the number line. When does the duck face the increasing numbers?**
Students	When the first factor is positive.
Teacher	**If the first factor is negative, the duck will face the decreasing numbers on the number line. When does the duck face the decreasing numbers?**
Students	When the first factor is negative.
Teacher	**So, which way will the duck face in this problem?**
Students	Decreasing.
Teacher	**Yes, the first factor is negative, so the duck faces the decreasing numbers.**

Students	(Place duck on zero. Make sure duck is facing decreasing numbers on the number line.)
Teacher	**Now, let's multiply. What is the second factor?**
Students	-2.
Teacher	**If the second factor is positive, the duck will move forward from its position. When does the duck move forward?**
Students	When the second factor is positive.
Teacher	**If the second factor is negative, the duck will move backward from its position. When does the duck move backward?**
Students	When the second factor is negative.
Teacher	**So, which direction should we move?**
Students	Backward.
Teacher	**Because the second factor is negative, we move backward. The second factor is -2 so we'll move by jumps of 2. Let's do that together. Count with me.**
Students	2, 4, 6.
Teacher	**So, our duck shows the product. What's the product?**
Students	6.
Teacher	**What's -3 times -2?**
Students	6.
Teacher	**Yes. -3 times -2 equals 6. Using this number line helps you understand what it means to multiply integers. How can you use the number line to multiply integers?**
Students	Start at zero. The duck faces increasing numbers with a positive factor and decreasing numbers with a negative factor. Then, the duck jumps the second factor by moving forward if it's a positive factor or backward if it's a negative factor.

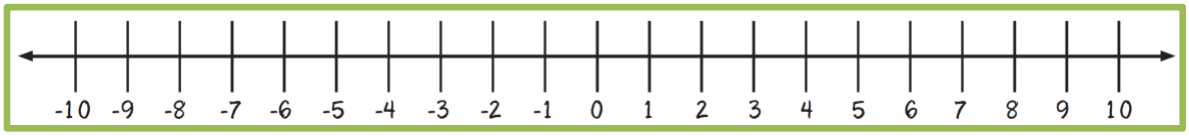
(2) Division with a Number Line

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A number line and a manipulative with a face (e.g., duck or dinosaur)

ROUTINE WITH NUMBER LINE



Teacher	**Let's divide integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on dividing with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we dividing?**
Students	__ divided by __.
Teacher	**So, let's start by thinking about the divisor. What's the divisor?**
Students	__.
Teacher	**Let's place the duck on the number line at zero. Where do we place the duck?**
Students	At zero.
Teacher	**If the divisor is positive, the duck will walk forward. When will the duck walk forward?**
Students	When the divisor is positive.

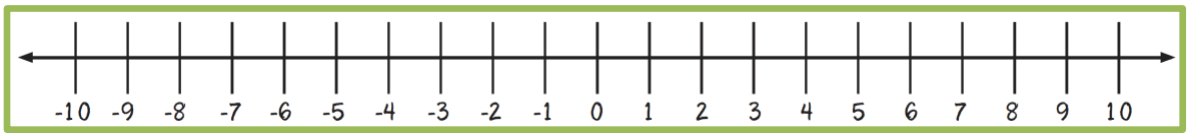
Teacher	**If the divisor is negative, the duck will walk backward. When will the duck walk backward?**
Students	When the divisor is negative.
Teacher	**Now, let's think about the dividend. The duck starts at zero and moves to the dividend. What's the dividend?**
Students	__.
Teacher	**The duck needs to move toward the dividend. If the duck will walk forward – because the divisor is positive – then face the duck toward the dividend. When do you face the duck toward the dividend?**
Students	When the divisor is positive and the duck will walk forward.
Teacher	**If the duck will walk backward – because the divisor is negative – then face the duck away from the dividend. When do you face the duck away from the dividend?**
Students	When the divisor is negative and the duck will walk backward.
Teacher	**So, which way will the duck face in this problem?**
Students	Toward the dividend/away from the dividend.
Teacher	**Yes, the dividend is positive/negative and the duck needs to walk forward/backward** (of the divisor)**, so the duck faces/doesn't face the dividend.**
Students	(Place the duck on zero. Make sure the duck is facing toward the dividend if the divisor is positive. Make sure the duck is facing away from the dividend if the divisor is negative.)
Teacher	**Now, let's divide. What's the divisor?**
Students	__.
Teacher	**So, the duck will jump the number of spaces in the divisor. If the divisor is 2, the ducks jumps in groups of 2. If the divisor is -5, the duck jumps in groups of 5. What would happen if the divisor is 10? How would the duck jump?**
Students	By 10.
Teacher	**So, the duck will jump the number of spaces in the divisor. And the jumps will be forward/backward because the divisor is positive/negative. Let's do that together. Count with me.**
Students	__, __, __, …
Teacher	**How many jumps did the duck make?**
Students	__.
Teacher	**__ is the quotient. Let's decide whether that's positive or negative. Is the duck facing the increasing numbers or decreasing numbers?**
Students	Increasing/decreasing.
Teacher	**If the duck faces the increasing numbers, then the quotient is positive. When is the quotient positive?**
Students	When the duck faces the increasing numbers.
Teacher	**If the duck faces the decreasing numbers, then the quotient is negative. When is the quotient negative?**
Students	When the duck faces the decreasing numbers.
Teacher	**What's the quotient?**

Students	__.
Teacher	**That's right. __ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Yes. __ divided by __ equals __. Using this number line helps you understand what it means to divide integers. How can you use the number line to divide integers?**
Students	Start at zero. If the divisor is positive, the duck will jump forward. If the divisor is negative, the duck will jump backward. We jump in groups of the divisor. That's the quotient. If the duck is facing the increasing numbers, the quotient is positive. If the duck is facing the decreasing numbers, the quotient is negative.

Example

12 ÷ (-3)

ROUTINE WITH NUMBER LINE



Teacher	**Let's divide integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on dividing with this number line.** (Show number line.) (Show problem.)
Teacher	**What numbers are we dividing?**
Students	12 divided by -3.
Teacher	**So, let's start by thinking about the divisor. What's the divisor?**
Students	-3.
Teacher	**Let's place the duck on the number line at zero. Where do we place the duck?**

Students	At zero.
Teacher	**If the divisor is positive, the duck will walk forward. When will the duck walk forward?**
Students	When the divisor is positive.
Teacher	**If the divisor is negative, the duck will walk backward. When will the duck walk backward?**
Students	When the divisor is negative.
Teacher	**Now, let's think about the dividend. The duck starts at zero and moves to the dividend. What's the dividend?**
Students	12.
Teacher	**The duck needs to move toward the dividend. If the duck will walk forward – because the divisor is positive – then face the duck toward the dividend. When do you face the duck toward the dividend?**
Students	When the divisor is positive and the duck will walk forward.
Teacher	**If the duck will walk backward – because the divisor is negative – then face the duck away from the dividend. When do you face the duck away from the dividend?**
Students	When the divisor is negative and the duck will walk backward.
Teacher	**So, which way will the duck face in this problem?**
Students	Away from the dividend.
Teacher	**Yes, the dividend is positive and the duck needs to walk backward because the divisor is negative, so the duck doesn't face the dividend.**
Students	(Place the duck on zero. Make sure the duck is facing away from the dividend if the divisor is negative.)
Teacher	**Now, let's divide. What's the divisor?**
Students	-3.
Teacher	**So, the duck will jump the number of spaces in the divisor. What's the divisor?**
Students	-3.
Teacher	**So, the duck will jump in groups of 3. And the jumps will be backward because the divisor is negative. Let's do that together. Count with me.**
Students	3, 6, 9, 12.
Teacher	**How many jumps did the duck make?**
Students	4.
Teacher	**The duck made 3 jumps. Is the duck facing the increasing numbers or decreasing numbers?**
Students	Decreasing.
Teacher	**If the duck faces the increasing numbers, then the quotient is positive. When is the quotient positive?**
Students	When the duck faces the increasing numbers.
Teacher	**If the duck faces the decreasing numbers, then the quotient is negative. When is the quotient negative?**
Students	When the duck faces the decreasing numbers.
Teacher	**What's the quotient?**

Students	-4.
Teacher	**That's right. 12 divided by -3 equals -4. Let's say that together.**
Students	12 divided by -3 equals -4.
Teacher	**Using this number line helps you understand what it means to divide integers. How can you use the number line to divide integers?**
Students	Start at zero. If the divisor is positive, the duck will jump forward. If the divisor is negative, the duck will jump backward. We jump in groups of the divisor. That's the quotient. If the duck is facing the increasing numbers, the quotient is positive. If the duck is facing the decreasing numbers, the quotient is negative.

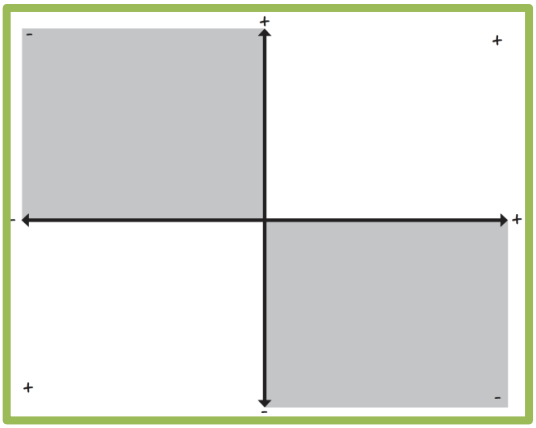
(3) Multiplication with Quadrant Mat and Cubes

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes

ROUTINE WITH QUADRANT MAT



Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this quadrant mat and these cubes.** (Show mat and cubes.)
Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.

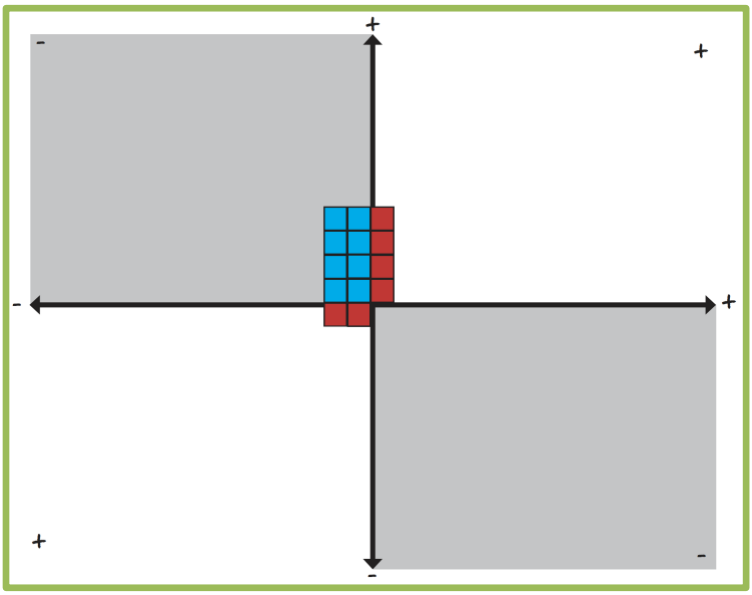
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat. (Show problem.)
Teacher	**What numbers are we multiplying?**
Students	__ times __.
Teacher	**So, let's start at the first factor. What's the first factor?**
Students	__.
Teacher	**Let's show the first factor with the cubes. We'll place the first factor on the horizontal axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the first factor?**
Students	Show __ cubes on the positive/negative side of the horizontal axis.
Teacher	**Yes, we'll show __ cubes on the positive/negative side of the horizontal axis.** (Show cubes.)
Teacher	**Now, let's multiply. What number do we multiply?**
Students	__.
Teacher	**Let's show the second factor with the cubes. We'll place the second factor on the vertical axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the second factor?**
Students	Show __ cubes on the positive/negative side of the vertical axis.
Teacher	**Yes, we'll show __ cubes on the positive/negative side of the vertical axis.** (Show cubes.)
Teacher	**Now, let's multiply. That means we multiply each of the cubes on the horizontal axis by each of the cubes on the vertical axis. Let me show you what I mean. On the horizontal axis, we have 1 cube. Let's multiply that cube by 1, 2, 3,… cubes on the vertical axis. I'll place the cubes in the rectangular area created by the multiplication. Where do I place the cubes?**
Students	In the rectangular area created by the multiplication. (Create area with cubes.)
Teacher	**Let's keep multiplying each cube on the horizontal axis until we've multiplied all the cubes.** (Create area with cubes.)
Teacher	**We've created an area with our multiplication. How many cubes are in that area?**
Students	__.
Teacher	**Is the area in a positive quadrant or negative quadrant?**
Students	Positive/negative.
Teacher	**So, what's __ times __?**
Students	__.
Teacher	**__ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Nice job! Using the quadrant mat and cubes helps you multiply integers. How can you use the quadrant mat and cubes to multiply integers?**

Students	Show the first factor on the horizontal axis. Show the second factor on the vertical axis. Multiply the cubes to create an area.

Example

-2 × (-4)

ROUTINE WITH QUADRANT MAT



Teacher	**Let's multiply integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on multiplying with this quadrant mat and these cubes.** (Show mat and cubes.)

Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat. (Show problem.)
Teacher	**What numbers are we multiplying?**
Students	-2 times 4.
Teacher	**So, let's start at the first factor. What's the first factor?**
Students	-2.
Teacher	**Let's show the first factor with the cubes. We'll place the first factor on the horizontal axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the first factor?**
Students	Show 2 cubes on the negative side of the horizontal axis.
Teacher	**Yes, we'll show 2 cubes on the negative side of the horizontal axis.** (Show cubes.)
Teacher	**Now, let's multiply. What number do we multiply?**
Students	4.
Teacher	**Let's show the second factor with the cubes. We'll place the second factor on the vertical axis on the positive side if the factor is positive and the negative side if the factor is negative. How do we show the second factor?**
Students	Show 4 cubes on the positive side of the vertical axis.
Teacher	**Yes, we'll show 4 cubes on the positive side of the vertical axis.** (Show cubes.)
Teacher	**Now, let's multiply. That means we multiply each of the cubes on the horizontal axis by each of the cubes on the vertical axis. Let me show you what I mean. On the horizontal axis, we have 1 cube. Let's multiply that cube by 1, 2, 3, 4 cubes on the vertical axis. I'll place the cubes in the rectangular area created by the multiplication. Where do I place the cubes?**
Students	In the rectangular area created by the multiplication. (Create area with cubes.)
Teacher	**Let's keep multiplying each cube on the horizontal axis until we've multiplied all the cubes. On the horizontal axis, we have a 2nd cube. Let's multiply that cube by 1, 2, 3, 4 cubes on the vertical axis. I'll place the cubes in the rectangular area created by the multiplication.** (Create area with cubes.)
Teacher	**We've created a rectangular area with our multiplication. How many cubes are in that area?**
Students	8.
Teacher	**Is the area in a positive quadrant or negative quadrant?**
Students	Negative.
Teacher	**So, what's -2 times 4?**
Students	-8.

Teacher	**-2 times 4 equals -8. Let's say that together.**
Students	-2 times 4 equals -8.
Teacher	**Nice job! Using the quadrant mat and cubes helps you multiply integers. How can you use the quadrant mat and cubes to multiply integers?**
Students	Show the first factor on the horizontal axis. Show the second factor on the vertical axis. Multiply the cubes to create an area.

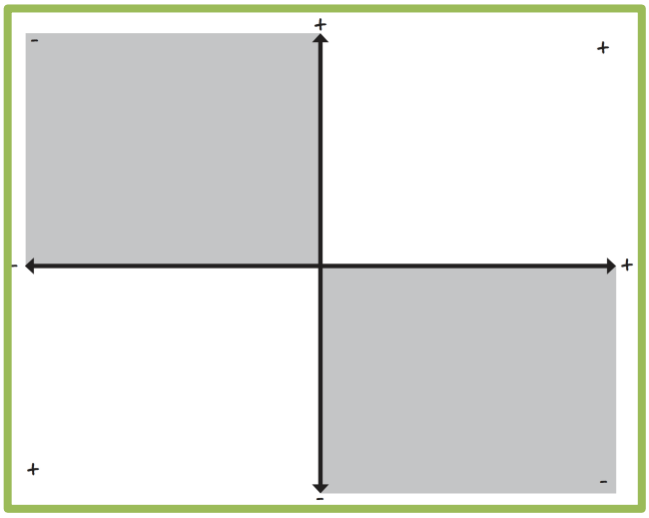
(4) Division with Quadrant Mat and Cubes

Routine

Materials:

- Module 19 Problem Sets
- Module 19 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like cubes

ROUTINE WITH QUADRANT MAT



Teacher	**Let's divide integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)
Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on dividing with this quadrant mat and these cubes.** (Show mat and cubes.)

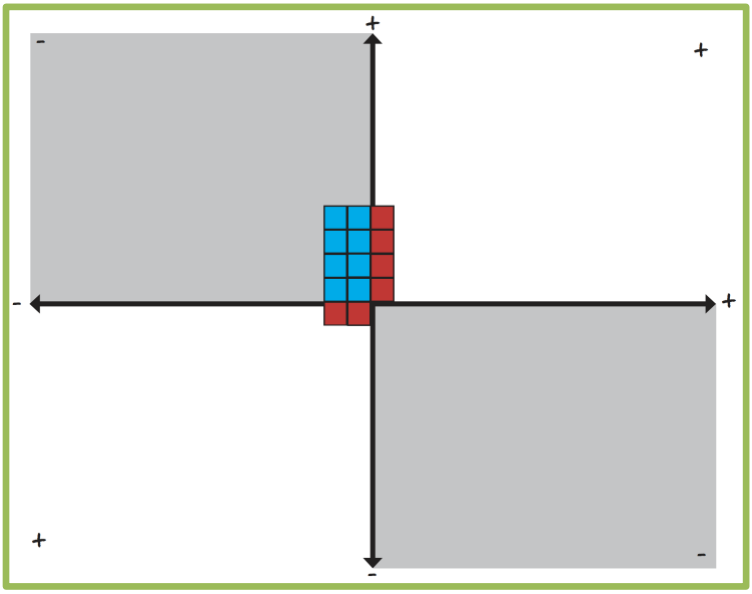
Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat. (Show problem.)
Teacher	**What numbers are we dividing?**
Students	__ divided by __.
Teacher	**So, let's start with the dividend. What's the dividend?**
Students	__.
Teacher	**Let's show the dividend with the cubes. We'll place the dividend in a positive quadrant if the dividend is positive. When do we place the dividend in a positive quadrant?**
Students	When the dividend is positive.
Teacher	**We'll place the dividend in a negative quadrant if the dividend is negative. When do we place the dividend in a negative quadrant?**
Students	When the dividend is negative.
Teacher	**Yes, we'll show __ cubes in a positive/negative quadrant.** (Show cubes.)
Teacher	**Now, let's divide. What number do we divide by? What's the divisor?**
Students	__.
Teacher	**Let's show the divisor with the cubes. We'll place the divisor on the positive side of an axis is the divisor is positive and the negative side of an axis if the divisor is negative. How do we show the divisor?**
Students	Show __ cubes on the positive/negative side of an axis. (Show cubes.)
Teacher	**You may have to move the dividend cubes to be near the divisor. For example, if you place the dividend cubes in the upper-right positive quadrant but the divisor is negative, you move the dividend cubes to the bottom-left positive quadrant. Do we need to move the dividend cubes?**
Students	Yes/no. (Move cubes if necessary.)
Teacher	**Now, let's divide. Let's see how many groups we can make with the divisor. So, we'll create groups of __** (divisor) **with the dividend. Let me show you what I mean. I can make 1 group. I'll place the cubes in a row by the divisor. Where do I place the cubes?**
Students	In a row by the divisor. (Show division into groups with cubes.)
Teacher	**Let's keep dividing until we've dividend all the cubes.** (Show division into groups with cubes.)
Teacher	**Now, let's determine our quotient by seeing how many groups we created. We created 1, 2, 3, … groups. How many?**
Students	__.

Teacher	**I'll place cubes on the axis to show the groups.** (Place cubes on axis.)
Teacher	**The cubes on the axis are the quotient. Is the quotient positive or negative? Look at the placement of the cubes on the axis.**
Students	Positive/negative.
Teacher	**So, what's __ divided by __?**
Students	__.
Teacher	**__ divided by __ equals __. Let's say that together.**
Students	__ divided by __ equals __.
Teacher	**Nice job! Using the quadrant mat and cubes helps you divide integers. How can you use the quadrant mat and cubes to divide integers?**
Students	Place the dividend cubes in a quadrant. Show the divisor cubes on one of the axes. Make groups of the divisor. Place the quotient cubes on the axis.

Example

-8 ÷ (-2)

ROUTINE WITH QUADRANT MAT



Teacher	**Let's divide integers. An integer is a positive or negative whole number. What's an integer?**
Students	A positive or negative whole number.
Teacher	**Let's think about a positive number. How do you know a number is positive?**
Students	It has a positive sign or it doesn't have a sign in front of the number.
Teacher	**We know a number is positive if the positive sign is directly in front of a number. The positive sign is a smaller plus sign.** (Draw +.)

Teacher	**We assume a number is positive if there is not a negative sign directly in front of a number. When do we assume a number is positive?**
Students	When there is not a negative sign directly in front of the number.
Teacher	**How do you know a number is negative?**
Students	It has a negative sign.
Teacher	**We know a number is negative if there is a negative sign directly in front of a number. The negative sign is a smaller minus sign.** (Draw -.)
Teacher	**Let's work on dividing with this quadrant mat and these cubes.** (Show mat and cubes.)
Teacher	**On the mat, we have a horizontal axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the horizontal axis?**
Students	Line across the mat.
Teacher	**On the mat, we have a vertical axis** (point). **This axis has a positive side** (point) **and negative side** (point). **What's the vertical axis?**
Students	Line up and down on the mat. (Show problem.)
Teacher	**What numbers are we dividing?**
Students	-8 divided by -2.
Teacher	**So, let's start with the dividend. What's the dividend?**
Students	-8.
Teacher	**Let's show the dividend with the cubes. We'll place the dividend in a positive quadrant if the dividend is positive. When do we place the dividend in a positive quadrant?**
Students	When the dividend is positive.
Teacher	**We'll place the dividend in a negative quadrant if the dividend is negative. When do we place the dividend in a negative quadrant?**
Students	When the dividend is negative.
Teacher	**Yes, we'll show 8 cubes in a negative quadrant.** (Show cubes.)
Teacher	**Now, let's divide. What number do we divide by? What's the divisor?**
Students	-2.
Teacher	**Let's show the divisor with the cubes. We'll place the divisor on the positive side of an axis is the divisor is positive and the negative side of an axis if the divisor is negative. How do we show the divisor?**
Students	Show 2 cubes on the negative side of an axis. (Show cubes.)
Teacher	**You may have to move the dividend cubes to be near the divisor. For example, if you place the dividend cubes in the upper-right positive quadrant but the divisor is negative, you move the dividend cubes to the bottom-left positive quadrant. Do we need to move the dividend cubes?**
Students	No.
Teacher	**Now, let's divide. Let's see how many groups we can make with the divisor. So, we'll create groups of 2 with the dividend. Let me show you what I**

	mean. I can make 1 group. I'll place the cubes in a row by the divisor. Where do I place the cubes?
Students	In a row by the divisor. (Show division into groups with cubes.)
Teacher	**Let's keep dividing until we've dividend all the cubes. I can make 2, 3, 4 groups.** (Show division into groups with cubes.)
Teacher	**Now, let's determine our quotient by seeing how many groups we created. We created 1, 2, 3, 4 groups. How many?**
Students	4.
Teacher	**I'll place 1, 2, 3, 4 cubes on the axis to show the groups.** (Place cubes on axis.)
Teacher	**The cubes on the axis are the quotient. Is the quotient positive or negative? Look at the placement of the cubes on the axis.**
Students	Positive.
Teacher	**So, what's -8 divided by -2?**
Students	4.
Teacher	**-8 divided by -2 equals 4. Let's say that together.**
Students	-8 divided by -2 equals 4.
Teacher	**Nice job! Using the quadrant mat and cubes helps you divide integers. How can you use the quadrant mat and cubes to divide integers?**
Students	Place the dividend cubes in a quadrant. Show the divisor cubes on one of the axes. Make groups of the divisor. Place the quotient cubes on the axis.

D. Problems for Use During Instruction

See Module 19 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 19 Vocabulary Cards.

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Module 19:

Multiplication and Division of Integers

Problem Sets

A. Positive integer times negative integer (20)
B. Negative integer times positive integer (20)
C. Negative integer times negative integer (20)
D. Positive integer divided by negative integer (20)
E. Negative integer divided by positive integer (20)
F. Negative integer divided by negative integer (20)

A.

$$3 \times (-8)$$

A.

$$6 \times (-4)$$

A.

$$7 \times (-2)$$

A.

$$5 \times (-10)$$

A.

$$9 \times (-3)$$

A.

$$2 \times (-5)$$

A.

$$6 \times (-3)$$

A.

$$7 \times (-8)$$

A.

$$4 \times (-9)$$

A.

$$11 \times (-12)$$

A.

$$5 \times (-5)$$

A.

$$11 \times (-10)$$

A.

$$8 \times (-4)$$

A.

$$13 \times (-8)$$

A.

$$7 \times (-4)$$

A.

$$12 \times (-3)$$

A.

$$9 \times (-6)$$

A.

$$4 \times (-5)$$

A.

$$2 \times (-3)$$

A.

$$0 \times (-9)$$

B.

$$(-6) \times 5$$

B.

$$(-3) \times 6$$

B.

$$(-9) \times 2$$

B.

$$(-4) \times 3$$

B.

$$(-7) \times 8$$

B.

$$(-5) \times 6$$

B.

$$(-7) \times 4$$

B.

$$(-6) \times 10$$

B.

$$(-3) \times 3$$

B.

$$(-10) \times 5$$

B.

$$(-2) \times 8$$

B.

$$(-7) \times 7$$

B.

$$(-11) \times 13$$

B.

$$(-12) \times 3$$

B.

$$(-14) \times 6$$

B.

$$(-11) \times 8$$

B.

$$(-15) \times 4$$

B.

$$(-8) \times 8$$

B.

$$(-2) \times 0$$

B.

$$(-8) \times 1$$

c.

$$(-2) \times (-3)$$

c.

$$(-6) \times (-5)$$

c.

$$(-8) \times (-4)$$

c.

$$(-9) \times (-9)$$

c.

$$(-5) \times (-7)$$

c.

$$(-4) \times (-6)$$

c.

$$(-11) \times (-6)$$

c.

$$(-3) \times (-4)$$

c.

$$(-8) \times (-10)$$

c.

$$(-7) \times (-12)$$

c.

$$(-9) \times (-4)$$

c.

$$(-8) \times (-6)$$

c.

$$(-12) \times (-9)$$

c.

$$(-3) \times (-15)$$

c.

$$(-16) \times (-2)$$

c.

$$(-7) \times (-11)$$

c.

$$(-12) \times (-4)$$

c.

$$(-13) \times (-3)$$

c.

$$(-12) \times (-5)$$

c.

$$(-16) \times (-2)$$

D.

$$9 \div (-3)$$

D.

$$6 \div (-2)$$

D.

$$5 \div (-5)$$

D.

$$14 \div (-2)$$

D.

$$45 \div (-5)$$

D.

$$18 \div (-2)$$

D.

$$49 \div (-7)$$

D.

$$54 \div (-6)$$

D.

$$21 \div (-3)$$

D.

$$32 \div (-4)$$

D.

$$18 \div (-3)$$

D.

$$40 \div (-5)$$

D.

$$12 \div (-6)$$

D.

$$48 \div (-8)$$

D.

$$72 \div (-9)$$

D.

$$63 \div (-7)$$

D.

$$16 \div (-8)$$

D.

$$20 \div (-5)$$

D.

$$10 \div (-2)$$

D.

$$18 \div (-6)$$

E.

$$(-28) \div 7$$

E.

$$(-40) \div 8$$

E.

$$(-12) \div 4$$

E.

$$(-14) \div 2$$

E.

$$(-49) \div 7$$

E.

$$(-63) \div 9$$

E.

$$(-20) \div 2$$

E.

$$(-16) \div 8$$

E.

$$(-18) \div 9$$

E.

$$(-14) \div 1$$

E.

$$(-21) \div 3$$

E.

$$(-48) \div 6$$

E.

$$(-44) \div 4$$

E.

$$(-81) \div 9$$

E.

$$(-56) \div 7$$

E.

$$(-25) \div 5$$

E.

$$(-12) \div 6$$

E.

$$(-64) \div 8$$

E.

$$(-15) \div 3$$

E.

$$(-72) \div 9$$

F.

$$(-18) \div (-9)$$

F.

$$(-12) \div (-3)$$

F.

$$(-24) \div (-6)$$

F.

$$(-40) \div (-5)$$

F.

$$(-72) \div (-8)$$

F.

$$(-36) \div (-6)$$

F.

$$(-20) \div (-4)$$

F.

$$(-70) \div (-7)$$

F.

$$(-21) \div (-3)$$

F.

$$(-45) \div (-9)$$

F.

$$(-27) \div (-3)$$

F.

$$(-15) \div (-5)$$

F.

$$(-16) \div (-4)$$

F.

$$(-10) \div (-5)$$

F.

$$(-30) \div (-6)$$

F.

$$(-32) \div (-8)$$

F.

$$(-99) \div (-9)$$

F.

$$(-24) \div (-2)$$

F.

$$(-36) \div (-3)$$

F.

$$(-27) \div (-9)$$

Module 19:

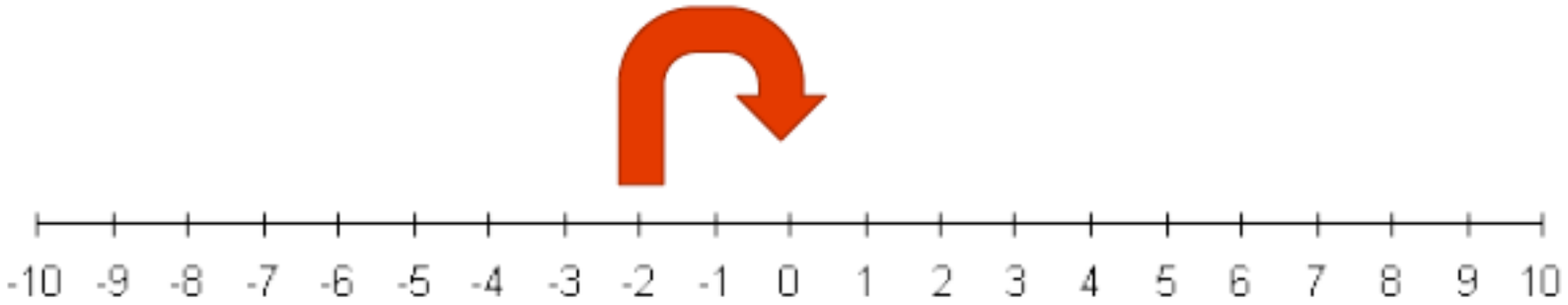
Multiplication and Division of Integers

Vocabulary Cards

absolute value
divide/division
dividend
divisor
factor
integer
multiply/multiplication
negative number
number line
opposites
positive number
product
quotient
zero pair

absolute value

The distance of a number from 0 on a number line.



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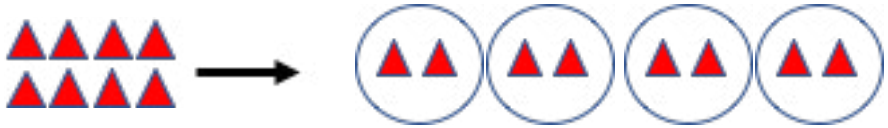
divide/division

To separate into equal groups.



8 ÷ 4 = 2

dividend

The number that is to be divided in a division problem.

16 ÷ 8 = 2

16 is the dividend

---

divisor

The number that the dividend is divided by.

16 ÷ 8 = 2

8 is the divisor

factor

A number that you multiply with another number to get the product.

$$2 \times 8 = 16$$

2 and 8 are the factors

---

integer

A positive or negative whole number.

-3 -2 -1 1 2 3

multiply/multiplication

The process of adding a number to itself a number of times.

$$4 \times 2 = 8$$

---

negative number

Any number less than 0.

-3 -2 -1

number line

A straight line with numbers placed at equal intervals along its length.

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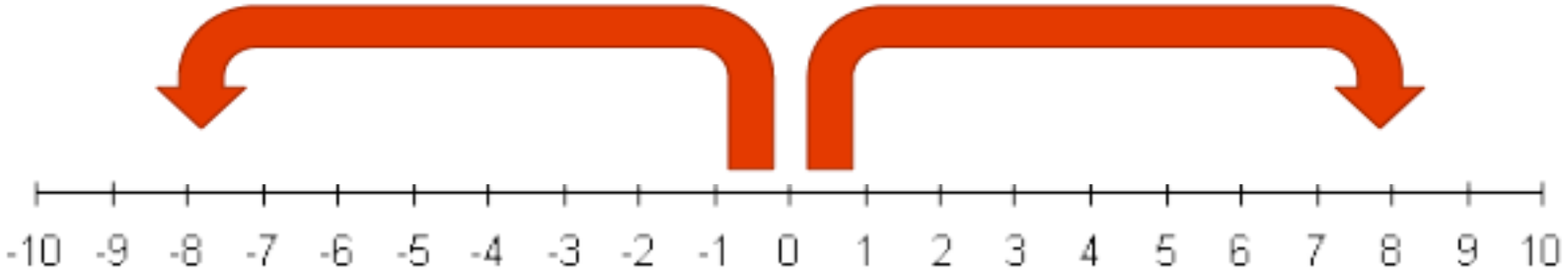
opposites

Two numbers that are equal distance from 0 on a number line.

-8 and 8 are opposites



positive number

Any number greater than 0.

1 2 3

---

product

The result of multiplying two or more factors.

$2 \times 8 = 16$

16 is the product

quotient

The number that results when one number is divided by another number.

$$16 \div 8 = 2$$

2 is the quotient

---

zero pair

A pair of numbers with a sum of 0.

$$-7 + 7 = 0$$