INCLUSION IN TEXAS

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 3-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

User Guide

I. Overview

Welcome to the *Instructional Routines for Mathematics Intervention User Guide*. These materials were created for Texas educators in partnership with the Texas Education Agency and Inclusion in Texas Network.

The goal of the *Instructional Routines for Mathematics* Intervention is to provide educators with a set of instructional materials to use when delivering mathematics intervention to students who experience difficulty. Each of the 23 Modules are focused on different mathematics content and are designed for implementation across grades 2 through 8.

The modules provide educators with easy-to-use materials for mathematics intervention. The aim is to provide educators with resources to address the diverse needs of students who experience mathematics difficulty. These materials can be implemented explicitly to improve mathematics outcomes for students.

Highlighted Module features:

- Step-by-step Routines for explicit teaching of the mathematics content.
- Vocabulary Cards with visuals that easily can be displayed in the classroom.
- Problem Sets with greater than 50 ready-to-use problems for each Module.

This User Guide includes the following sections:

These materials are designed to be easily accessed and utilized. We look forward to hearing your success stories and feedback. Enjoy!

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II. Materials

The table below provides a summary of the 23 Modules with the applicable Texas Essential Knowledge and Skills (TEKS) for each grade level.

Module Number	Module Title	Module Description	Texas Essential Knowledge and Skills (TEKS) Applicable Grade Levels						
			2	3	4	5	6	7	8
1	Place Value	Describe place value of whole and rational numbers	2(A) 2(B)	2(A) 2(B) 2(C)	2(A) 2(B) 2(D) 4(G)	2(C)			
2	Comparison	Compare whole and rational numbers with greater than, less than, or equal to	2(D) 2(E)	2(D) 3(H)	2(C) 2(F) 3(D)	2(B)			2(D)
3	Representing Fractions	Show fractions with the length, area, and set models	3(A) 3(B) 3(C) 3(D)	3(A) 3(B) 3(C) 3(D) 3(E) 3(F) 3(G)	3(B) 3(C) 3(D) 3(G)		2(D) 4(E) 4(F) 4(G) 5(C)		
4	Concepts of Addition	Describe addition as (a) combining sets and (b) joining to a set	4(B) 4(C)	4(A) 4(B)					
5	Addition of Whole Numbers	Add multi-digit numbers using (a) standard algorithm and (b) partial sums	4(B) 4(C)	4(A) 4(B) 5(A)	4(A)	3(A)		2	
6	Addition of Rational Numbers	Add rational numbers with like denominators and unlike denominators			3(A) 3(E) 3(F) 4(A)	3(A) 3(H) 3(K)		3(A) 3(B)	
7	Concepts of Subtraction	Describe subtraction as (a) separating from a set and (b) comparing	4(B) 4(C)	4(A) 4(B)					
8	Subtraction of Whole Numbers	Subtract multi-digit numbers using (a) standard algorithm and (b) adding up	4(B) 4(C)	4(A) 4(B) 5(A)	4(A)	3(A)		2	
9	Subtraction of Rational Numbers	Subtract rational numbers with like denominators and unlike denominators			3(E) 3(F) 4(A)	3(A) 3(H) 3(K)		3(A) 3(B)	

Module Number	Module Title	Module Description	Texas Essential Knowledge and Skills (TEKS) Applicable Grade Levels						
			2	3	4	5	6	7	8
10	Concepts of Multiplication	Describe multiplication as (a) equal groups and (b) comparison	6(A)	4(D) 4(E) 4(F) 4(H) 5(B) 5(D)					
11	Multiplication of Whole Numbers	Multiply multi-digit numbers using (a) standard algorithm and (b) partial products/array		4(D) 4(E) 4(F) 4(G) 4(K) 5(C)	4(B) 4(C) 4(D) 4(H)	3(A) 3(B)			
12	Multiplication of Rational Numbers	Multiply fractions with an emphasis on conceptual understanding				3(A) 3(D) 3(E) 3(I)	3(A) 3(B) 3(E)	2 3(A) 3(B)	
13	Concepts of Division	Describe division as (a) partitive and (b) measurement	6(B)	4(H) 4(I) 4(J) 5(D)					
14	Division of Whole Numbers	Divide multi-digit numbers using (a) standard algorithm and (b) partial quotients		4(H) 4(I) 4(J) 4(K)	4(E) 4(F) 4(H)	3(A) 3(C)			
15	Division of Rational Numbers	Divide fractions with an emphasis on conceptual understanding				3(A) 3(F) 3(G) 3(J) 3(L)	3(A) 3(E)	2 3(A) 3(B)	
16	Representing Decimals	Show decimals using proportional and non-proportional materials			2(E) 2(F) 2(G) 2(H) 3(G)	2(A) 2(B)	4(E) 4(F) 4(G) 5(C)		2(C)
17	Integers	Understand positive and negative integers					2(B) 2(C)		
18	Addition and Subtraction of Integers	Add and subtract positive and negative integers					3(C) 3(D)		
19	Multiplication and Division of Integers	Multiply and divide positive and negative integers					3(D)		

Module Number	Module Title	Module Description	Texas Essential Knowledge and Skills (TEKS) Applicable Grade Levels						
			2	3	4	5	6	7	8
20	Functions and Ordered Pairs	Describe functions, ordered pairs, and graphing related to functions		5(E)	5(B)	4(C) 4(D)	4(A) 7	7(A)	5(A) 5(C) 5(G)
21	Ratios, Proportions, Rates, and Percentages	Represent ratios, proportions, rates, and percentages					4(B) 4(C) 4(D) 4(E) 4(F) 4(G) 4(H) 5(A) 5(B) 5(C)	4(A) 4(B) 4(C) 4(D) 4(E)	5(A) 5(B) 5(C) 5(D) 5(E) 5(F) 5(G) 5(H) 5(I)
22	Representing Expressions and Equations	Describe order of operations, representing expressions, and representing equations				4(E) 4(F)	6(A) 6(B) 6(C) 7(B) 7(C) 7(D)	7	
23	Solving Equations	Solve (a) single-step equations with one variable, (b) multi-step equations with one variable, and (c) equations with variables on both sides					9(A) 9(B) 9(C) 10(A) 10(B)	10(A) 10(B) 10(C) 11(A) 11(B) 11(C)	8(A) 8(B) 8(C) 9

Each of the 23 Modules includes the following components:

1. Routines
2. Vocabulary Cards
3. Problem Sets

Section IV describes the Routines, Vocabulary Cards, and Problem Sets in detail.

III. Effective Mathematics Teaching

As with all mathematics teaching, when implementing the Modules, educators need to deliver effective mathematics instruction. Effective mathematics instruction includes (but is not limited to) the following 3 strategies, which are described in detail in this section:

- Explicit Instruction
- Formal Mathematics Language
- Multiple Representations

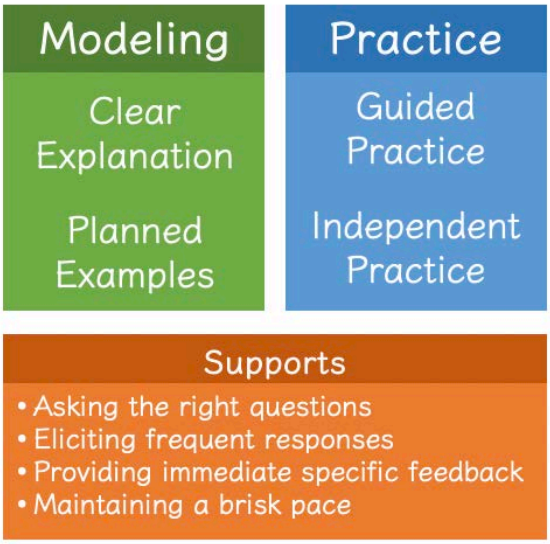
Explicit Instruction

Explicit instruction is defined as "a way of teaching where the educator selects an important objective, specifies the learning outcome, designs structured instructional experiences, explains directly, models the skills being taught, and provides scaffolded practice to help a student achieve mastery" (Kearns, 2018).

Explicit instruction is an evidence-based practice that benefits all students, particularly those identified with disabilities and learning difficulties. For this reason, explicit instruction should be an integral component of any lesson.

The primary components of explicit instruction are:

- **Modeling**: facilitated by the educator.
- **Practice:** involves the student and educator.
- **Supports:** involves an ongoing dialogue between the student and educator. Supports are used during modeling *and* during practice.



Modeling

Modeling prepares students to complete a mathematics skill successfully. Modeling includes two primary components: Clear Explanations and Planned Examples.

Clear Explanations

- Provide a short statement about the lesson's goals and importance.
- Explicitly model the steps for completing the task or solving the problem.
- Incorporate vocabulary and concise mathematics language (see next section).
- Pre-select examples depending on students' exposure to the content.
- Adjust modeling based on students' needs.

Sample Modeling from Module 1: Place Value	
Clear Explanations	**Dialogue**
Goal and importance	*Let's work on composing and decomposing numbers. Composing means to make numbers. What does composing mean?* *Today, we'll compose numbers with these Base-10 blocks.*
Explicitly model steps with concise mathematics language *(Note: **bolded words** represent concise mathematics language)*	*When we read numbers, we read numbers by **period**. A period is each group of digits separated by a comma or the **decimal point**.* *Our common **periods** include the **millions, thousands, ones,** then **thousandths**. What are the common **periods**?* *Let's write __ in **expanded notation**.* *Let's start with the **greatest place value**. What's the **greatest place value** in this number?*

Planned Examples

- Plan examples in a purposeful way prior to the lesson.
- Ask important questions.
- Vary examples: include worked examples of problems solved correctly *and* problems solved incorrectly, non-examples, and open-ended examples.

For planned examples, educators should consider which problems to include from the Problem Sets provided in each Module. Educators also need to create worked examples that are most appropriate for their students. Educators should plan ahead of each lesson and consider a variety of planned examples.

Practice

Practice is intended to provide multiple opportunities for students to *practice* the learned mathematics concepts. Students with disabilities and learning difficulties require additional practice to master new concepts and skills. To ensure students receive sufficient learning opportunities, both Guided Practice and Independent Practice should be included in every lesson.

Guided Practice

- Consists of the educator and students working together to solve problems.
- Can take place at a group table, with the educator and students working together.
- Can take place with the educator at the whiteboard and students at their desks.
- Provides supports to promote understanding and to encourage students' success.
- Includes the use of questioning and mathematics tools (e.g., manipulatives).
- Provides a scaffolded release of responsibility from modeling to independent practice.

Independent Practice

- Consists of students working independently under the guidance of the educator.
- Allows for the educator to provide feedback and answer questions.
- Provides a way to monitor the level of support needed for students to understand.
- Should not be reserved only for homework.

Supports

Students should actively participate in the lessons through supports. During modeling *and* practice, educators should attend to the following four supports, which should be included in every lesson:

Ask High-level and Low-Level Questions

- Ask a combination of high-level and low-level questions to evaluate students' understanding.
- Promote conceptual understanding and reasoning with high-level questions.
- Check for procedural understanding and increase participation with low-level questions.
- Ask a question every 30-60 seconds during modeling to promote active engagement.
- Examples:
    - *How could you explain dividing to a friend?* (high-level)
    - *What is a quotient?* (low-level)

Elicit Frequent Responses

- Engage students frequently by eliciting responses every 30-60 seconds during modeling.
- Provide a variety of ways for students to respond
    - Orally, in writing, chorally, pictures, whiteboard, gesturing, etc.

Provide Immediate Affirmative and Corrective Feedback

- Foster confidence to encourage students with low self-esteem and anxiety.
- Provide feedback immediately and as often as possible.
- Make affirmative feedback specific and related to the mathematics concept.
- Use questioning and encouragement when providing corrective feedback.
- Examples:
    - *I noticed you are using the fraction tiles to demonstrate three-fifths.* (affirmative)
    - *Can you tell your neighbor how you solved the problem?* (corrective)
    - *Can you explain the steps you followed to solve this expression?* (corrective)

Maintain a Brisk Pace

- Plan and organize prior to the lesson.
- Consider any needed materials and technology prior to the lesson.
- Consider any planned examples, including worked examples and non-examples.
- Consider which seating charts and/or student groupings will optimize learning.
- Be knowledgeable about the material and prepared to demonstrate effective modeling.

How Do I Use Explicit Instruction with the Modules?

The components of modeling are provided in the Routines for each Module. When planning for these lessons, you should consider additional examples such as worked examples with problems solved correctly and incorrectly and non-examples. Guided practice can be "modeled" from Routines using the Vocabulary Cards and Problem Sets. Educators should think about the independent practice experiences that will optimize learning for their students. Remember, independent practice should provide an opportunity for students to practice the learned skills independently under the guidance of the educator. Independent practice should not be reserved for homework.

As you plan to incorporate explicit instruction into your teaching of the Modules, consider using the explicit instruction framework from the National Center for Intensive Intervention, displayed below.

MODELING	PRACTICE
Clear explanation	Guided
Planned examples	Independent

Supporting Practices
Asking the right questions
Eliciting frequent responses
Providing immediate specific feedback
Maintaining a brisk pace

Formal Mathematics Language

As educators use explicit instruction, it is important to focus on formal mathematics language. Formal mathematics language refers to the precise mathematics terms used to describe mathematics concepts and procedures.

Examples of formal mathematics vocabulary terms include *product, angle,* and *denominator.* In contrast, informal mathematics language consists of words like *answer, corner,* and *bottom number in the fraction.*

Students are responsible for a tremendous amount of mathematics language at each grade level. At grade 3, students are exposed to over 300 different mathematics terms in their mathematics textbook glossaries. By grade 6, that number grows to over 500 terms. Therefore, it is necessary to have an explicit focus on the language of mathematics.

In addition to the sheer number of terms, the mathematics language often is complex. Mathematics terms are challenging for students, especially those experiencing learning difficulties because of:

- Technical terms that students have never seen (e.g., *perimeter*)
- Multiple meanings in mathematics and English (e.g., *degree*)
- Multiple meanings in mathematics (e.g., *quarter*)
- Multiple meanings across context areas (e.g., *base* in science vs. *base* in math)
- Vocabulary terms with multiple words (e.g., *rectangular prism*)
- Homonyms (e.g., *have* and *half)*
- Similarities to or differences from native language words (e.g. *quarter* vs. *cuarto*)

To promote students' understanding of formal mathematics language, educators should:

1. Use Formal Mathematics Vocabulary Terms During Instruction
    - Use formal mathematics vocabulary terms over informal phrases during every lesson.
    - Align the terms used with those presented in textbooks, videos, and on assessments.
    - Frequently expose students to formal terms in preparation for activities and tests.
    - Explicitly teach mathematics vocabulary terms to ensure students understand.
    - Examples:
        - **Module 2: Comparison**: say *greater than* instead of *bigger.*
        - **Module 6: Addition of Rational Numbers**: say *sum* instead of *answer.*
        - **Module 22: Representing Expressions and Equations**: say *variable* instead of *x.*

2. Use Similar and Related Terms Correctly and Precisely
    - Be correct, precise, and specific when using closely related mathematics terms.
    - Reflect on which formal vocabulary terms to explicitly teach to students.
    - Select terms that directly align with students' language skills, knowledge, and familiarity with the mathematics content.
    - Explicitly teach vocabulary words with a specific mathematics meaning.

3. Plan for Language Use Prior to Instruction
    - Consider language use (as well as students' language use) prior to instruction.
    - Avoid using limited or informal language that does not prepare students for success.
    - Present formal mathematics language from textbooks, assessments, and videos during instruction to support students' long-term learning.

4. Include Explicit Vocabulary Activities in Instruction
    - Directly teach the Vocabulary Cards from each Module to students.
    - Provide meaningful practice opportunities for students to use the Vocabulary Cards.
    - Include vocabulary activities to ensure students actively practice using terms.
    - Consider concept maps, word walls, and student dictionaries of mathematics terms.
    - Use mnemonic devices to access students' prior knowledge.
    - Offer multiple exposures of mathematics terms to build fluency.
    - Consider games to increase students' motivation.

5. Hold Students Accountable
    - Provide opportunities for students to listen to and read formal mathematics language.
    - Create experiences for students to speak and write using formal mathematics language.
    - Focus on using formal language to describe mathematics concepts and procedures.

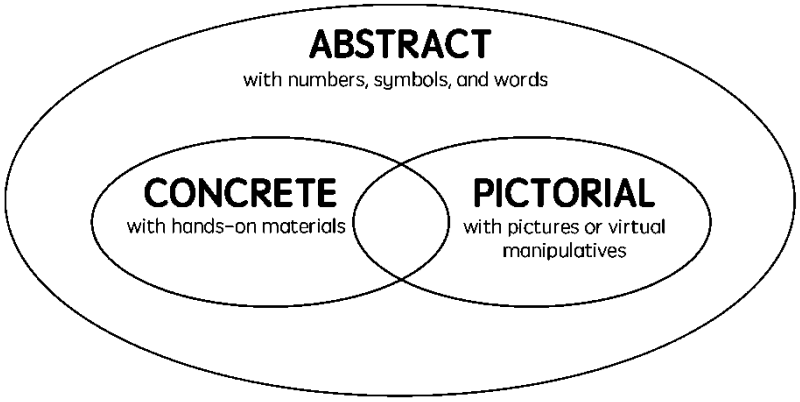
How Do I Use Formal Mathematics Language with the Modules?

The table below offers a few examples of how educators can translate their informal mathematics language to formal and precise mathematics language when teaching the Modules related to fractions. "*Say this*" refers to the formal mathematics language educators should incorporate into lessons. "*Instead of that*" refers to the informal mathematics language that does not help students to develop a conceptual understanding of the mathematics content. As you teach each Module, use formal mathematics language whenever possible.

"Say this"	"Instead of that"
Numerator and denominator	*Top number and bottom number*
Find an equivalent fraction	*Reduce the fraction*
Demonstrate process within Base-10	*Move the decimal point over*
Two-thirds	*2 over 3*
This fraction is a number	*Numbers in the fraction*
Three and four tenths	*Three point four*

Multiple Representations

In addition to using explicit instruction and formal mathematics language, educators should incorporate multiple representations into the mathematics Modules to support students with learning difficulties.



In this User Guide, multiple representations includes the abstract, concrete, and pictorial forms of mathematics. The goal is to utilize the abstract, concrete, and/or pictorial supports as necessary to promote students' deeper understanding of the mathematics concepts and procedures. Some students may need extra practice using the concrete forms; others may require additional pictorial supports to access the abstract. Ultimately, students with and without learning difficulties benefit from using a combination of these three supports. The three primary components of multiple representations include:

Abstract

- Consists of numbers, symbols, and words.
- Reflects the typical view of mathematics (e.g., 42 + 102 = 144).
- Often requires the concrete and pictorial to support students' understanding.

Concrete

- Refers to three-dimensional, hands-on materials and objects that students can touch.
- Includes hands-on formal manipulatives like fraction bars, algebra tiles, geoboards, etc.
- Includes hands-on manipulatives that are less formal (e.g., straws, paper clips).

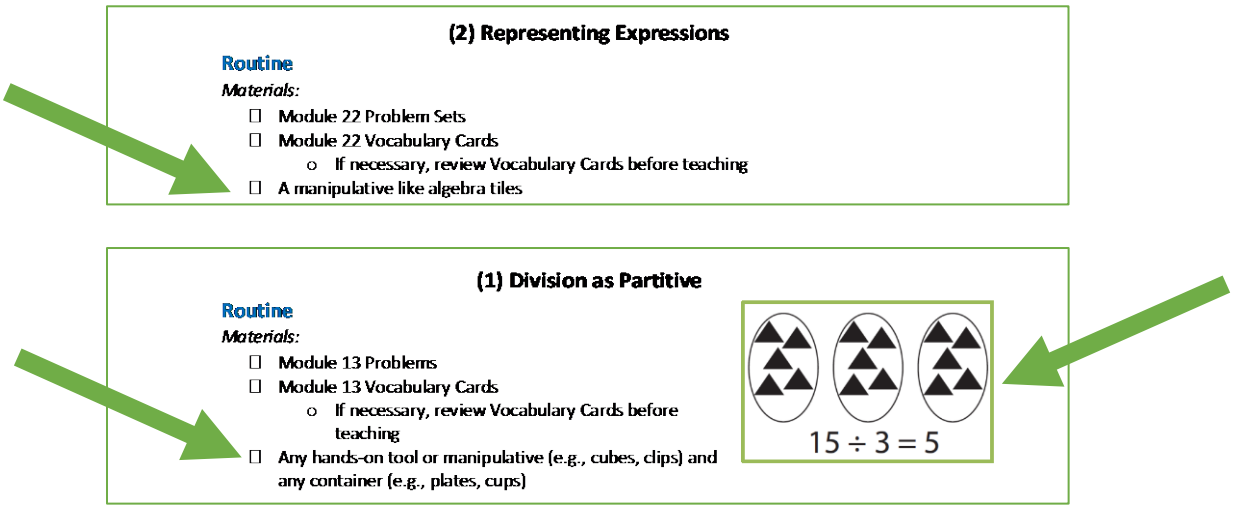
Pictorial/Virtual

- Includes two-dimensional pictures, images, or virtual manipulatives.
- Often refers to the semi-concrete or representational.
- Includes visuals within textbooks or workbooks, in educator and student drawings, and on high-stakes standardized assessments.
- Includes graphic organizers that help students understand mathematics concepts.
- Includes the use of virtual manipulatives.

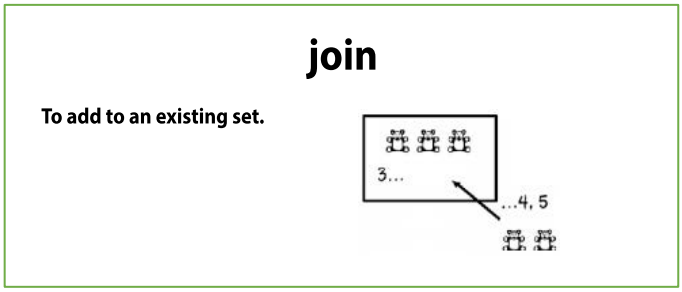
How Do I Use Multiple Representations with the Modules?

Abstract and pictorial representations are included throughout the Routines and Vocabulary Cards. The materials section of each Routine provides suggested concrete manipulatives to support students' understanding of the concepts and procedures (see below). Supplement the lessons with additional concrete, pictorial/virtual, and abstract representations based on the specific needs of your students.

Here are examples of multiple representations embedded within the Routines.



Here are examples of multiple representations embedded within the Vocabulary Cards.



subtract/subtraction

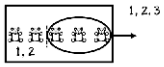
To compare two sets or to take away from a set.

To compare two sets

5 – 3 = 2

To take away from a set

5 – 3 = 2

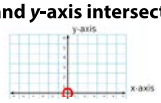


area

The number of square units that covers a closed figure.

origin

A point where the *x*-axis and *y*-axis intersect. The origin has the coordinates (0, 0).



number line

A straight line with numbers placed at equal intervals along its length.

IV. How to use the Routines, Vocabulary Cards, and Problem Sets

Each Module includes:

- Routines
- Problem Sets
- Vocabulary Cards

Routines

At the beginning of each Routine, you will see *Section A: Important Vocabulary with Definitions. Section A* highlights the mathematics vocabulary that will be explicitly used in the Routine and/or the vocabulary to review before using the Routine.

A. Important Vocabulary with Definitions

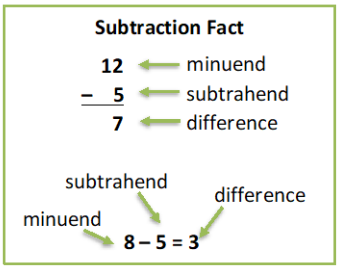
Term	Definition
compare	To find the difference between two sets.
difference	The result of subtracting one number from another number.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
minuend	The number from which another number is subtracted.
minus sign	The symbol that tells you to subtract.
separate	To start with a set and take away from that set.
subtract/subtraction	To compare two sets or to separate from a set.
subtrahend	The number to be subtracted.

After *Section A*, you will see *Section B: Background Information. Section B* provides brief background information on the mathematics content of the Routine. Often important vocabulary or the sequence of learning the mathematics content is described. The information in this section is for educators, but can be shared with students.

B. Background Information

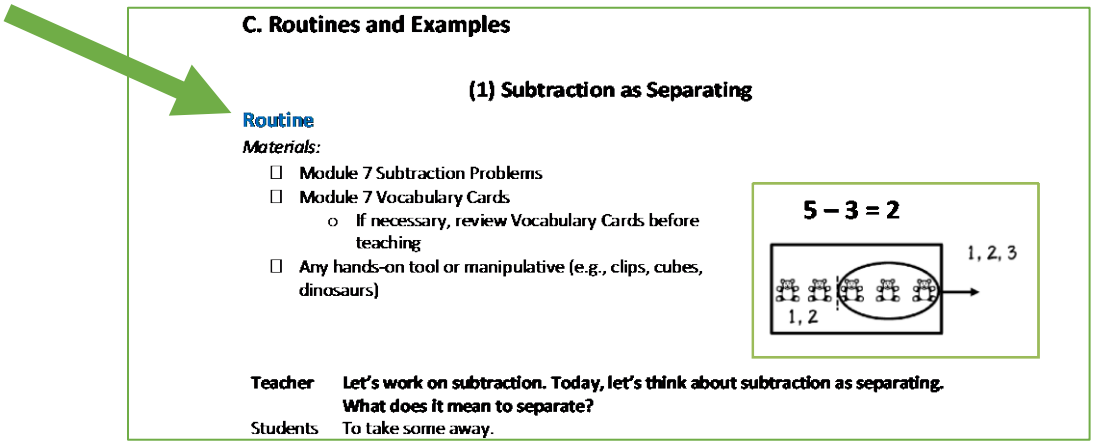
Students need to learn two concepts of subtraction: (1) subtraction as separating from a set and (2) subtraction as comparison for a difference. Typically, students first learn about subtraction as separating from a set. Then, students learn about comparing two sets for a difference.

For learning the concepts of subtraction, we recommend using *mathematics facts*. We define a subtraction mathematics fact as a single- or double-digit minuend less than 19 and a single-digit subtrahend. The subtrahend is subtracted from the minuend for a difference. You may present subtraction facts vertically or horizontally.



The main portion of each Routine is *Section C: Routines and Examples.* In each Module, there are several Routines. As an example, the picture below shows the Routine for *Subtraction as Separating*. This include a list of materials necessary for the Routine. In some cases hands-on tools or manipulatives that can be used along with the Routine. Remember that virtual manipulatives may be used as well.

Following the materials, is a description of how to teach a specific skill. This is the **Routine.** Each Routine outlines educator dialogue (**in bold)** and planned student responses (unbolded). Teachers are not required to read the Routines verbatim. Instead, read the Routine before teaching to become familiar with the content and its delivery.



C. Routines and Examples

(1) Subtraction as Separating

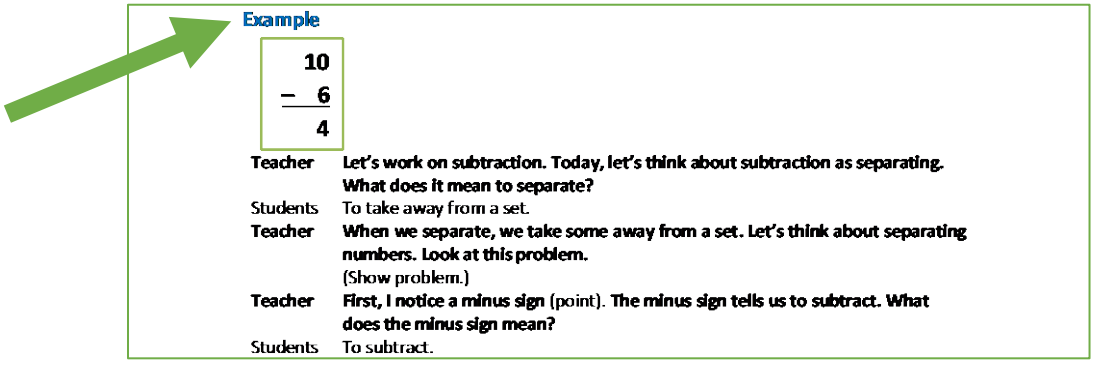
Routine

Materials:

- ☐ Module 7 Subtraction Problems
- ☐ Module 7 Vocabulary Cards
  - o If necessary, review Vocabulary Cards before teaching
- ☐ Any hands-on tool or manipulative (e.g., clips, cubes, dinosaurs)

Teacher	**Let's work on subtraction. Today, let's think about subtraction as separating. What does it mean to separate?**
Students	To take some away.

After most Routines, there is an **Example.** The Example shows how to use the Routine with a specific mathematics problem.

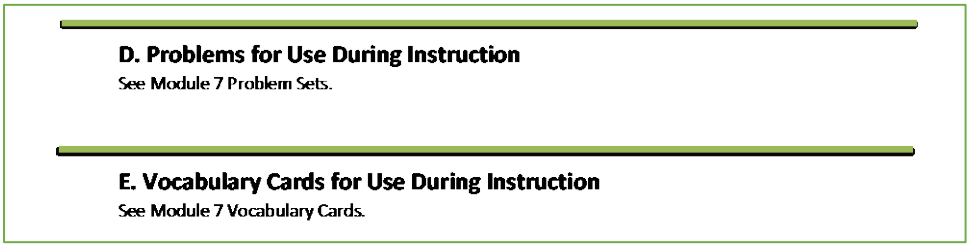


Example

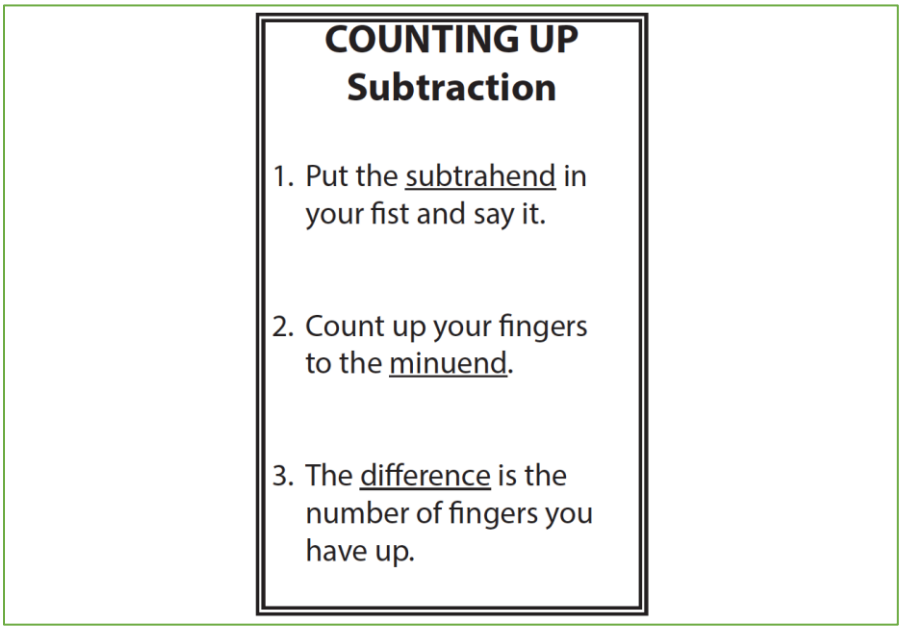
$$\begin{array}{r} 10 \\ -\ 6 \\ \hline 4 \end{array}$$

Teacher	**Let's work on subtraction. Today, let's think about subtraction as separating. What does it mean to separate?**
Students	To take away from a set.
Teacher	**When we separate, we take some away from a set. Let's think about separating numbers. Look at this problem.** (Show problem.)
Teacher	**First, I notice a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.

Following all the Routines and Examples, there is a reference to *Section D: Problems for Use During Instruction* and *Section E: Vocabulary Cards for Use During Instruction.*

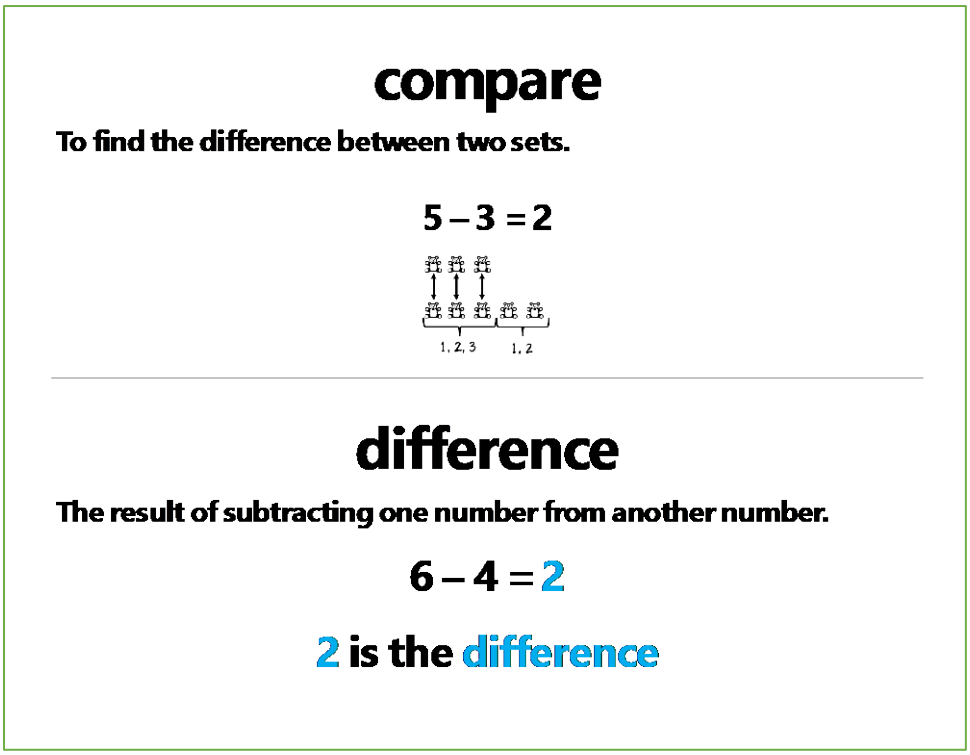


Several Modules include *Section F: Supplementary Materials.* Below is an example of a Counting Poster used to accompany a subtraction Routine.



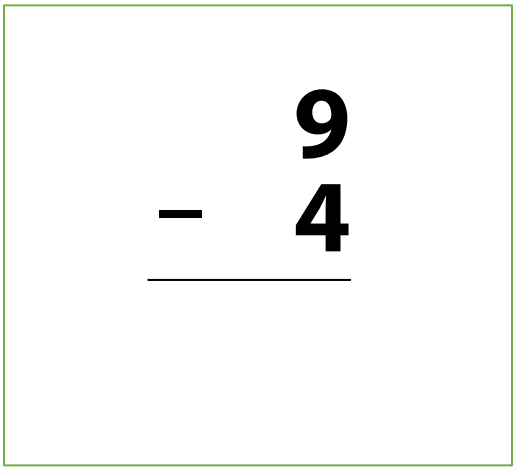
Vocabulary Cards

Vocabulary Cards are available for each of the vocabulary terms listed at the beginning of a Routine. Educators may choose to place these Vocabulary Cards on a mathematics word wall or instruct students to add the Vocabulary Cards to their mathematics glossaries or journals. The example below shows the Vocabulary Cards for the terms *compare* and *difference.*

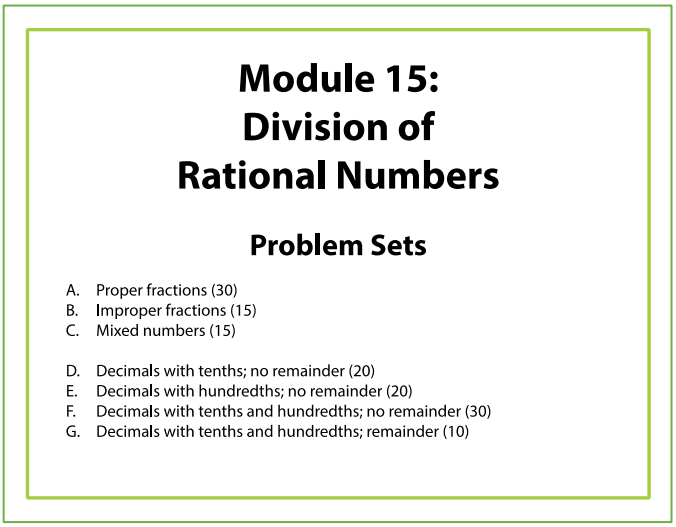


Problem Sets

Problems to accompany each Module's Routine. Problems are purposefully placed one per page. Educators may choose to show these problems on their screen or document camera. Educators also can print out the Problem Sets for use during modeling, guided practice, and independent practice.

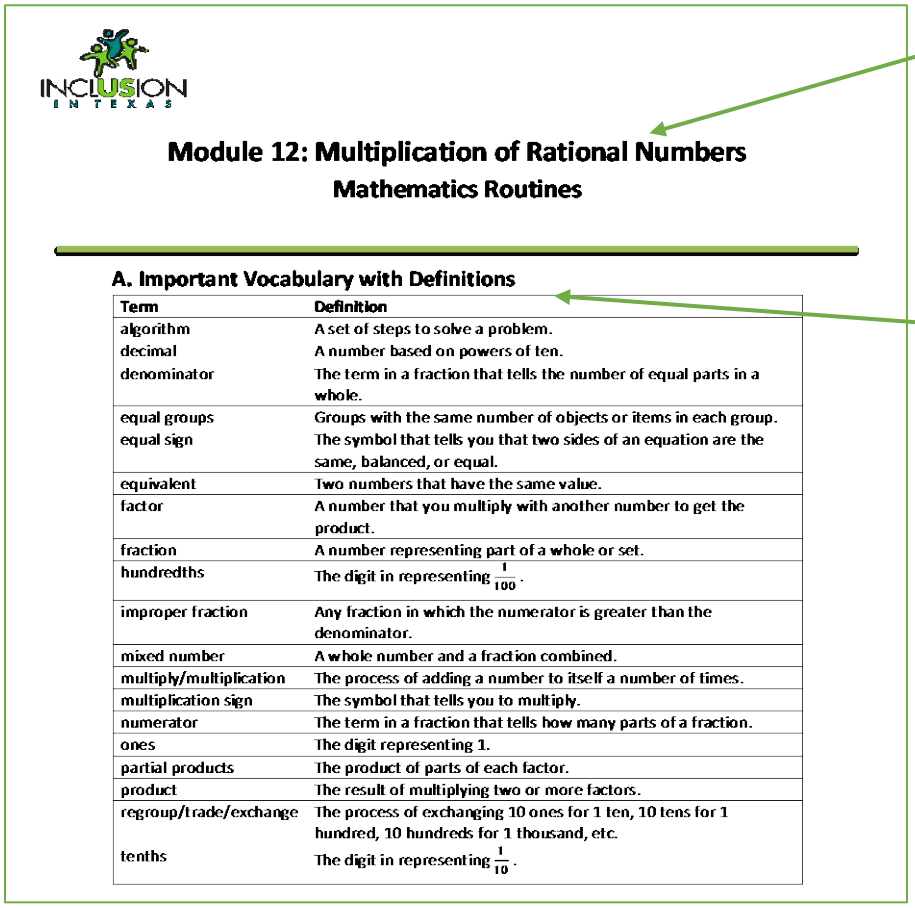


Many of the Problem Sets have a variety of choices (see example below). The number in parentheses after the description indicates the number of problems for that type. For example, there are 30 problems for Proper fractions. Educators do not need to use all of these Problem Sets. The Problem Sets provide variety and choice during instruction. Educators should view the Problem Sets in advance of the lesson to select the problems that are most appropriate for their students.



VI. Constructing a Lesson: An Example

This section provides an example of how to use a Module in mathematics intervention.



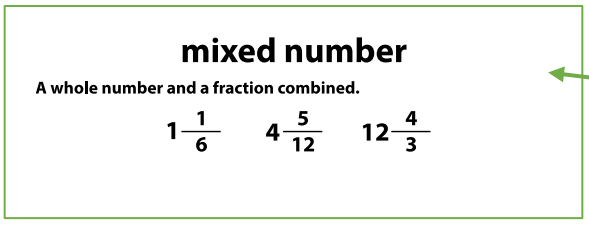
Module 12: Multiplication of Rational Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A set of steps to solve a problem.
decimal	A number based on powers of ten.
denominator	The term in a fraction that tells the number of equal parts in a whole.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
equivalent	Two numbers that have the same value.
factor	A number that you multiply with another number to get the product.
fraction	A number representing part of a whole or set.
hundredths	The digit in representing $\frac{1}{100}$.
improper fraction	Any fraction in which the numerator is greater than the denominator.
mixed number	A whole number and a fraction combined.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
numerator	The term in a fraction that tells how many parts of a fraction.
ones	The digit representing 1.
partial products	The product of parts of each factor.
product	The result of multiplying two or more factors.
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.
tenths	The digit in representing $\frac{1}{10}$.

Module 12 is about multiplication of rational numbers.

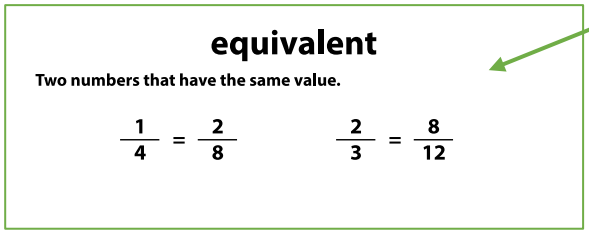
This is a list of important mathematics vocabulary for Module 12. Prior to the lesson, review the vocabulary and identify terms students may need to learn or practice.



mixed number

A whole number and a fraction combined.

$1\frac{1}{6}$ $4\frac{5}{12}$ $12\frac{4}{3}$

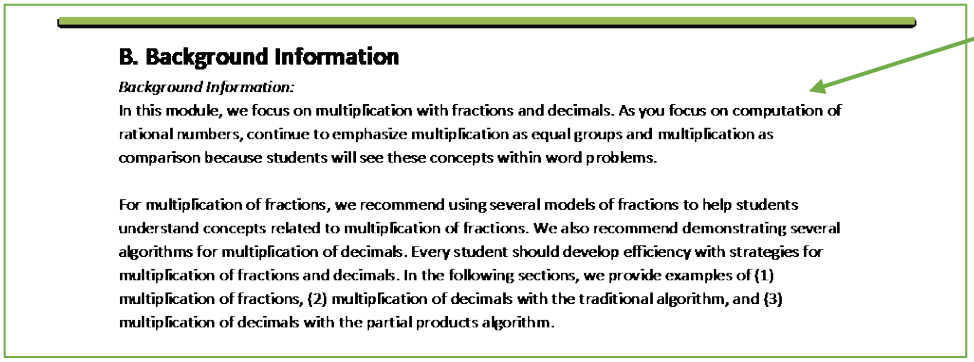


equivalent

Two numbers that have the same value.

$\frac{1}{4} = \frac{2}{8}$ $\frac{2}{3} = \frac{8}{12}$

Use the Vocabulary Cards to teach or review important vocabulary terms with students.



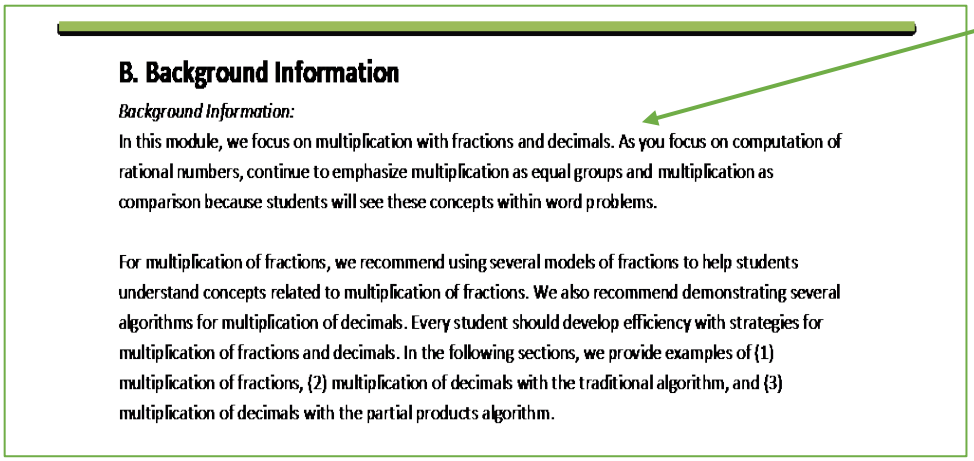
B. Background Information

Background Information:

In this module, we focus on multiplication with fractions and decimals. As you focus on computation of rational numbers, continue to emphasize multiplication as equal groups and multiplication as comparison because students will see these concepts within word problems.

For multiplication of fractions, we recommend using several models of fractions to help students understand concepts related to multiplication of fractions. We also recommend demonstrating several algorithms for multiplication of decimals. Every student should develop efficiency with strategies for multiplication of fractions and decimals. In the following sections, we provide examples of (1) multiplication of fractions, (2) multiplication of decimals with the traditional algorithm, and (3) multiplication of decimals with the partial products algorithm.

This is background information about the Module.



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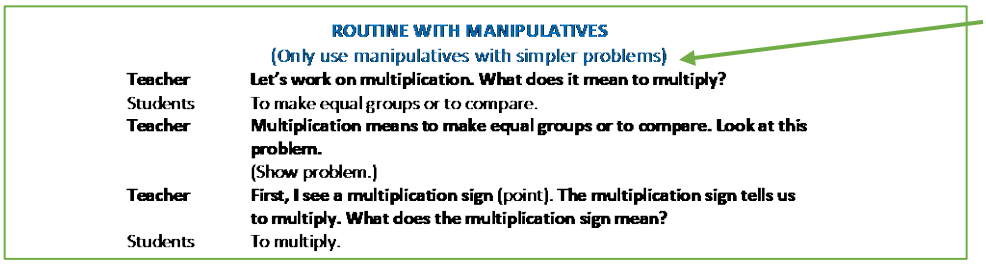
This Module has 3 separate Routines:
(1) Multiplication of Fractions
(2) Multiplication of Decimals with Traditional Algorithm
(3) Multiplication of Decimals with Partial Products



ROUTINE WITH MANIPULATIVES

(Only use manipulatives with simpler problems)

Teacher	**Let's work on multiplication. What does it mean to multiply?**
Students	To make equal groups or to compare.
Teacher	**Multiplication means to make equal groups or to compare. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.

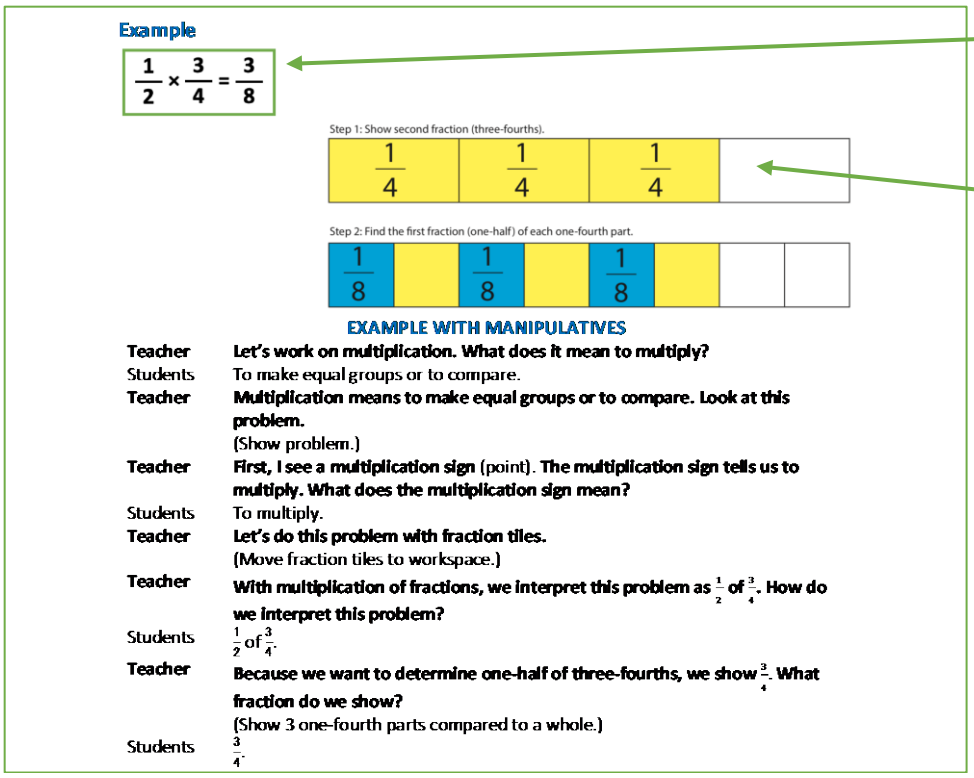
The Routine provides a description of how to use explicit instruction to model a mathematics skill.

Teacher	**Let's do this problem with fraction tiles.** (Move fraction tiles to workspace.)
Teacher	**With multiplication of fractions, we interpret this problem as __ (first fraction) of __ (second fraction). How do we interpret this problem?**
Students	__ of __.
Teacher	**We want to determine __ (first fraction) of __ (second fraction). If you wanted to determine half of 8, you would show 8 and then find half of that amount. The same works with fractions. We'll show the second fraction (or factor) and then find the first fraction of the second fraction. Which fraction will we show?**
Students	Second fraction.
Teacher	**So, let's show the second fraction with the fraction tiles.** (Show second fraction with fraction tiles.)
Teacher	**Now, let's find __ (first fraction) of __ (second fraction). There are several ways to do this, but an easy way is to find __ (first fraction) of each one-__ (second fraction denominator) part. Let's focus on one-__ part at a time. What should we focus on?**
Students	One-__ part.
Teacher	**Let's just think about this one-__ part (second fraction denominator). What's __ (first fraction) of this part?**
Students	__.
Teacher	**If that's hard to answer, think about it this way. What's __ (first fraction) times one-__ (second fraction denominator)?**
Students	__.
Teacher	**__ (first fraction) of this one-__ part (second fraction denominator) would be __. Let's place that/those fraction tiles on top of the one-__ part.** (Place fraction tiles.)
Teacher	**Now, I do that again for each one-__ part. I find __ (first fraction) of each one-__ part.** (Place fraction tiles.)
Teacher	**We're multiplying by finding __ (first fraction) of each of the one-__ parts. How are we multiplying?**
Students	Finding __ (first fraction) of each of the one-__ parts.
Teacher	**We've determined __ (first fraction) of each of the one-__ parts with the fraction tiles, these are our partial products. What are these?**
Students	Partial products.
Teacher	**Let's add the partial products to determine the final product. What should we add?**
Students	The partial products.
Teacher	**We have __ plus __ plus … . That equals __. Say that with me.**
Students	__.
Teacher	**So, __ (first fraction) of __ (second fraction) equals __. What's the product?**

The **bolded text** is for the educator. The planned students responses are un-bolded.

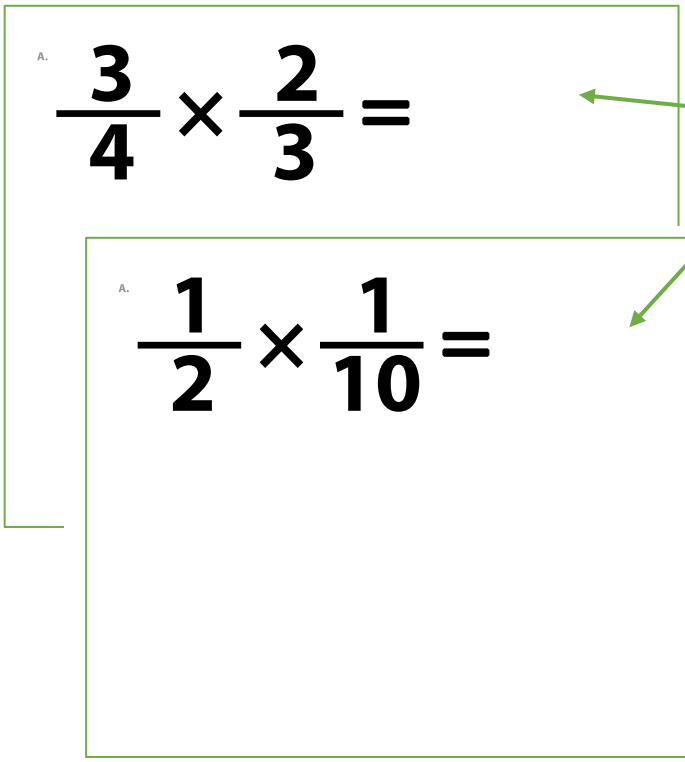
As you use the Routine, make sure you are (a) asking the right questions, (b) eliciting student responses frequently, and (c) providing immediate, specific feedback. The Routine includes suggested questions. Teachers may go beyond the written Routine based on intervention with students.

The blank (___) indicates when to fill in information about a specific problem.



This Routine is followed by an Example.

Example snapshot of the manipulatives used to solve this specific problem.



When teaching, choose problems from the Problem Sets.

The Problem Sets can be used during modeling, guided practice, and independent practice by:

- showing the Problem Sets on a document camera or tablet
- printing the problems on paper, or
- using student transcription to a whiteboard.

VII. Glossary of Vocabulary Terms

Vocabulary Term	Definition	In Which Module(s)?
absolute value	The distance of a number from 0 on a number line.	17, 18, 19
add/addition	To put amounts together to find the sum or to increase a set.	4, 5, 6
addend	Any numbers that are added together.	4, 5, 6, 18
algorithm	A procedure or description of steps that can be used to solve a problem.	5, 6, 8, 9, 11, 12, 14, 15
area	The number of square units that covers a closed figure.	10, 11
array	A set of objects, pictures, or numbers arranged in columns and rows.	10, 11
base	A number that is multiplied by an exponent.	22, 23
coefficient	A number that is multiplied by a variable.	21, 22, 23
commutative property (of multiplication)	Two factors can be multiplied in any order.	11
compare (comparison)	To examine differences between numbers, quantities, or values to decide if one quantity is greater than, less than, or equal to another quantity.	2
compare (subtraction)	To find the difference between two sets.	7, 8, 9
compose	To make a number.	1
computation	The action used to solve a problem.	5, 6, 8, 9, 11, 14, 15
constant	A term that does not change; a number on its own.	21, 22, 23
coordinate plane	A two-dimensional plane formed at the intersection of the *x*-axis and *y*-axis.	20
decimal	A number based on powers of ten.	1, 6, 9, 12, 15, 16
decimal point	A dot used to separate ones from tenths in a number or dollars from cents.	1, 16
decompose	To break apart by place value.	1
denominator	The term in a fraction that tells the number of equal parts in a whole.	2, 3, 6, 9, 12, 15, 21
difference	The result of subtracting one number from another number.	7, 8, 9, 18
digit	A symbol used to show numbers.	1, 2
divide/division	To separate into equal groups or among groups.	13, 14, 15, 19
dividend	The number to be divided.	13, 14, 15, 19
division sign	The symbol that tells you to divide.	13, 14, 15
divisor	The number the dividend is divided by.	13, 14, 15, 19
equal	When the number, quantity, or value on the left side of the equal sign is the same as the number, quantity, or value on the right side of the equal sign.	2
equal groups	Groups with the same number of objects or items in each group.	10, 11, 12, 13, 14, 15
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.	2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 21

Vocabulary Term	Definition	In Which Module(s)?
equation	A mathematical statement that two expressions are the same or equal; must have an equal sign.	20, 22, 23
equivalent	Two numbers that have the same value.	2, 3, 6, 9, 12
equivalent fractions	Fractions that have different numerators and denominators that represent the same value or proportion of the whole.	21
equivalent ratios	Ratios that have the same fractional number, value, or measure.	21
estimate	To give an approximate value rather than an exact answer.	1
expanded form	Writing a number to show the place value of each digit.	1
exponent	The power to which a number is raised.	22, 23
expression	A combination of variables, numbers, and/or operations that represents a mathematical relationship; does not have an equal sign.	20, 22, 23
factor	A number you multiply with another number to get the product.	10, 11, 12, 19
fraction	A number representing part of a whole or set.	2, 3, 6, 9, 12, 15, 21
function	A relationship between two quantities in which every input corresponds to exactly one output.	20
function table	A table that displays a set of inputs and outputs in such a way that each input has a unique output.	20
greater than	When the number, quantity, or value on one side of the equal sign is larger than the number, quantity, or value on the other side of the equal sign.	2
grouping	A combination of variables, numbers, and/or operations grouped together in parentheses or brackets.	22, 23
hundred thousands	The digit representing 100,000.	1
hundreds	The digit representing 100.	1, 2, 16
hundreds column	The column with digits in the hundreds place.	5, 8, 11, 14
hundredths	The digit in representing $\frac{1}{100}$.	1, 6, 9, 12, 15, 16
improper fraction	Any fraction in which the numerator is greater than or equal to the denominator.	3, 6, 9, 12, 15, 21
inequality	An algebraic relation showing that a quantity is greater or less than another quantity.	22, 23
input variable	The *x* of an equation; the information put in to find the output.	20
integer	A positive or negative whole number.	17, 18, 19
join	To add to an existing set.	4, 5, 6
least common multiple	The common multiple with the least value.	6, 9, 12, 15, 21
less than	When the number, quantity, or value on one side of the equal sign is smaller than the number, quantity, or value on the other side of the equal sign.	2

Vocabulary Term	Definition	In Which Module(s)?
like fractions	Fractions that have the same denominator.	21
like terms	Terms that have the same variable or constant and can be combined.	22, 23
lowest terms	A fraction is reduced to lowest terms when there is no number other than 1 that will evenly divide into both the numerator and denominator.	21
minuend	The number from which another number is subtracted.	7, 8, 9, 18
minus sign	The symbol that tells you to subtract.	7, 8, 9
mixed number	A whole number and a fraction combined.	3, 6, 9, 12, 15, 21
multiple	The product of a number and any integer.	6, 9, 21
multiplication sign	The symbol that tells you to multiply.	10, 11, 12
multiply/multiplication	The process of adding a number to itself a number of times.	10, 11, 12, 19
negative number	Any number less than 0.	17, 18, 19
number line	A straight line with numbers placed at equal intervals along its length.	2, 17, 18, 19
numerator	The term in a fraction that tells how many parts of a fraction.	2, 3, 6, 9, 12, 15, 21
ones	The digit representing 1.	1, 2, 6, 9, 12, 15, 16
ones column	The column with digits in the ones place.	5, 8, 11, 14
operator	A symbol (+, –, × ÷) that represents a mathematical operation.	22, 23
opposites	Two numbers that are equal distance from 0 on a number line.	17, 18, 19
ordered pair	A pair of numbers used to locate a point on a coordinate plane.	20
origin	A point where the *x*-axis and *y*-axis intersect. The origin has the coordinates (0, 0).	20
output variable	The *y* of an equation; the information gained after the input is plugged into an equation.	20
partial products	The product of parts of each factor.	10, 11, 12
partitive division	To share equally among groups.	13
percentage	A rate of an amount per hundred.	21
period	A group of three digits with each group separated by a comma.	1
place value	The value of a digit depending on its place in a number.	1, 2, 16
plus sign	The symbol that tells you to add.	4, 5, 6
positive number	Any number greater than 0.	17, 18, 19
product	The result of multiplying two or more factors.	10, 11, 12, 19
proper fraction	A fraction where the numerator is less than the denominator.	3, 21
proportion	An equation that states that two ratios are equal.	21

Vocabulary Term	Definition	In Which Module(s)?
quadrant	The *x*- and *y*-axes divide the coordinate plane into four regions called quadrants.	20
quotative division	To measure objects into groups.	13
quotient	The result when one number is divided by another number.	13, 14, 15, 19
rate	A comparison of two quantities that have different units of measure.	21
ratio	A comparison of two quantities that have the same unit of measure.	21
rational number	Any number that can be written as a fraction.	2
reciprocal	The reciprocal of a number is 1 divided by that number.	15
regroup/trade/exchange	The process of exchanging 10 ones for 1 ten, 10 tens for 1 hundred, 10 hundreds for 1 thousand, etc.	5, 6, 8, 9, 11, 12, 14, 15
remainder	The amount remaining in a division problem.	14, 15
rounding	A process that tells which place value a number is closest to.	1
separate	To start with a set and take away from that set.	7, 8, 9
standard form	A way to write numbers using digits.	1
subtract/subtraction	To compare two sets or to take away from a set.	7, 8, 9
subtrahend	The number to be subtracted.	7, 8, 9, 18
sum	The result of adding two or more numbers or the total number when you combine sets.	4, 5, 6, 18
ten thousands	The digit representing 10,000.	1
tens	The digit representing 10.	1, 2, 16
tens column	The column with digits in the tens place.	5, 8, 11, 14
tenths	The digit in representing $\frac{1}{10}$.	1, 6, 9, 12, 15, 16
term	A single number or a variable, or numbers or variables multiplied together.	22, 23
thousands	The digit representing 1,000.	1, 2, 16
thousandths	The digit in representing $\frac{1}{1000}$.	1, 16
together	To combine sets or numbers.	4, 5, 6
unit rate	A ratio that is written as a number to one.	21
variable	A symbol for an unknown value, which is usually represented by a letter.	22, 23
word form	The form of a number that uses written words.	1
x-axis	The horizontal number line on a coordinate plane.	20
y-axis	The vertical number line on a coordinate plane.	20