INCLUSION IN TEXAS

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA
Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 8

Subtraction of Whole Numbers

Module 8: Subtraction of Whole Numbers
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
algorithm	A procedure or description of steps that can be used to solve a problem.
compare	To find the difference between two sets.
computation	The action used to solve a problem.
difference	The result of subtracting one number from another number.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
hundreds column	The column with digits in the hundreds place.
minuend	The number from which another number is subtracted.
minus sign	The symbol that tells you to subtract.
ones column	The column with digits in the ones place.
regroup/trade/exchange	The process of exchanging 1 ten for 10 ones, 1 hundred for 10 tens, 1 thousand for 10 hundreds, etc.
separate	To start with a set and take away from that set.
subtract/subtraction	To compare two sets or to separate from a set.
subtrahend	The number to be subtracted.
tens column	The column with digits in the tens place.

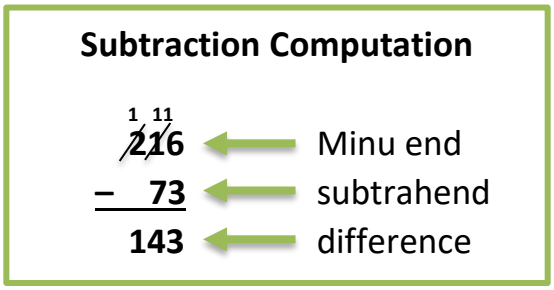
B. Background Information

Background Information:

If your focus is on the conceptual understanding of subtraction, see *Module 7: Concepts of Subtraction*. This module, *Module 8*, focuses on subtraction computation of whole numbers. As you focus on computation, continue to emphasize subtraction as separating and subtraction as comparing because students will see these concepts within word problems.

For learning computation with subtraction, we recommend presenting problems vertically. Some students may require explicit instruction on translating a horizontal problem (e.g., 124 – 83) to the vertical presentation (see below). Depending upon the algorithm, leave enough space above or below the problem for students to complete their written work.

Every student should develop efficiency with a subtraction computation strategy. In the following sections, we provide examples of (1) subtraction with a traditional algorithm – no regrouping, (2) subtraction with a traditional algorithm – regrouping, (3) subtraction with partial differences algorithm, and (4) subtraction with an adding up algorithm. Teachers should understand different algorithms and help students to develop competency with at least one algorithm.



C. Routines and Examples

(1) Subtraction with Traditional Algorithm – No Regrouping

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT – 2-DIGIT: ROUTINE WITH MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with Base-10 blocks.** (Move Base-10 blocks to workspace.)
Teacher	**With our Base-10 blocks, the rods represent tens. What do the rods represent?**
Students	Tens.
Teacher	**With our Base-10 blocks, the units represent ones. What do the units represent?**

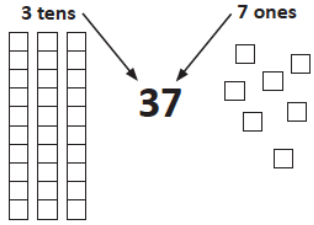
Students Ones.

Teacher **Our minuend is __. What's our minuend?**

Students __.

Teacher **Let's show this minuend by showing __ tens and __ ones.**

(Show with Base-10 blocks.)



Teacher **How many?**

Students __.

Teacher **Our subtrahend is __. What's our subtrahend?**

Students __.

Teacher **Let's subtract the subtrahend. In this example, we'll think about subtraction as separating, but we could also think about subtraction as comparing. What do we subtract?**

Students Subtrahend.

Teacher **What's the subtrahend in this problem?**

Students __.

Teacher **Let's first subtract the ones of the subtrahend. We separate __ ones from the minuend. Do we have enough ones in the minuend to subtract __ ones?**

Students Yes.

Teacher **We have enough ones. Let's separate or take away __ ones.**

(Remove ones.)

Teacher **Now, let's subtract the tens of the subtrahend. We separate __ tens from the minuend. Do we have enough tens in the minuend to subtract __ tens?**

Students Yes.

Teacher **We have enough tens. Let's separate or take away __ tens.**

(Remove tens.)

Teacher **Let's count to learn the difference.**

(Count the tens, then count the ones.)

Teacher **That means __ minus __ equals __. Let's say that together.**

Students __ minus __ equals __.

Teacher **Let's say it together again.**

Students __ minus __ equals __.

Teacher **So, if you have a set of __ and separate __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The result of subtracting a subtrahend from a minuend.

Teacher **What does it mean to separate?**
Students To take away.
Teacher **How could you explain separating to a friend?**
Students We started with a set of Base-10 blocks. We separated the ones and tens of the subtrahend. We counted to learn the difference.
Teacher **What's another way we could have solved this problem?**
Students We could have compared two sets.

2-DIGIT – 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**
Students To separate or compare.
Teacher **Subtraction means to separate from a set or to compare two sets. Look at this problem.**
(Show problem.)
Teacher **First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students To subtract.
Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**
(Draw vertical lines to separate place value columns.)
Teacher **Now, we start by subtracting the ones. What should we subtract first?**
Students The ones.
Teacher **Which ones do we subtract?**
Students __ minus __.
Teacher **Do you have enough ones to subtract __ ones?**
Students Yes.
Teacher **You have enough ones to subtract or take away __ ones. We don't have to regroup. What's __ minus __?**
(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher **How many ones are remaining?**
Students __.
Teacher **Yes! There are __ ones. Let's write __ below the equal line.**
(Write.)
Teacher **Now, let's subtract the tens. Which tens do we subtract?**
Students __ minus __.
Teacher **Do you have enough tens to subtract __ tens?**
Students Yes.

Teacher	**You have enough tens to subtract or take away __ tens. We don't have to regroup. What's __ minus __?** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher	**How many tens are remaining?**
Students	__.
Teacher	**There are __ tens. Let's write __ below the equal line.** (Write.)
Teacher	**So, what's __ minus __?**
Students	__.
Teacher	**That's right. __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**So, if you have a set of __ and subtract __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the ones and then we subtracted the tens to learn the difference between two numbers.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

```
 458
−  26
 432
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Example

3-DIGIT – 2-DIGIT: EXAMPLE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.

Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens and the tens from the hundreds. Let's draw a vertical line between the ones column and the tens column. Then, draw a vertical line between the tens column and the hundreds column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by subtracting the ones. What should we subtract first?**
Students	The ones.
Teacher	**Which ones do we subtract?**
Students	8 minus 6.
Teacher	**If you have 8 ones, can you subtract 6 ones?**
Students	Yes.
Teacher	**You have enough ones to subtract 6 ones. Let's subtract 8 minus 6.** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher	**How many ones are remaining?**
Students	2.
Teacher	**Yes! There are 2 ones remaining. Let's write 2 under the equal line in the ones place.** (Write 2.)
Teacher	**Now, let's subtract the tens. Which tens do we subtract?**
Students	5 minus 2.
Teacher	**If you have 5 tens, can you subtract 2 tens?**
Students	Yes.
Teacher	**Great. You have enough tens to subtract 2 tens. What's 5 minus 2?** (If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)
Teacher	**How many tens are remaining?**
Students	3.
Teacher	**There are 3 tens. Let's write 3 under the equal line in the tens place.** (Write 3.)
Teacher	**Now, let's subtract the hundreds. Which hundreds do we subtract?**
Students	4 minus nothing or 0.
Teacher	**If you have 4 hundreds, can you subtract 0?**
Students	Yes.
Teacher	**You can subtract 4 minus 0. What's 4 minus 0?**
Students	4.
Teacher	(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __:**

__, __, __. See Counting Up poster at the end of Module 7 for more information.)

Teacher **How many hundreds are remaining?**

Students 4.

Teacher **There are 4 hundreds. Let's write 4 under the equal line in the hundreds place.**

(Write 4.)

Teacher **What's 458 minus 26?**

Students 432.

Teacher **That's right. 458 minus 26 equals 432. Let's say that together.**

Students 458 minus 26 equals 432.

Teacher **So, if you have a set of 458 and separate 26, the difference is 432. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The result of subtracting a subtrahend from a minuend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students We subtracted the ones. Then, we subtracted the tens. Then, we subtracted the hundreds to learn the difference between 458 and 26.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

(2) Subtraction with Traditional Algorithm – Regrouping

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like Base-10 blocks or unifix cubes
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT – 2-DIGIT: ROUTINE WITH MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**

Students To separate or compare.

Teacher **Subtraction means to separate from a set or compare two sets. Look at this problem.**

(Show problem.)

Teacher **First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**

Students To subtract.

Teacher **Let's do this problem with Base-10 blocks.**

(Move Base-10 blocks to workspace.)

Teacher **With our Base-10 blocks, the rods represent tens. What do the rods represent?**

Students Tens.

Teacher **With our Base-10 blocks, the units represent ones. What do the units represent?**

Students Ones.

Teacher **Our minuend is __. What's our minuend?**

Students __.

Teacher **Let's show the minuend by showing __ tens and __ ones.**

(Show with Base-10 blocks.)

Teacher **How many?**

Students __.

Teacher **Now, we separate the subtrahend from the minuend. What's our subtrahend?**

Students __.

Teacher **Let's first subtract the ones of the subtrahend. We separate __ ones from the minuend. How many ones?**

Students __.

Teacher **Look at the minuend. Do we have enough ones in the minuend to subtract __ ones?**

Students No!

Teacher **We do not have enough ones. That means we have to regroup. To regroup, we take 1 ten and regroup/trade/exchange the 1 ten for 10 ones. Let's do that together.**

(Show 1 ten is equivalent to 10 ones.)

Teacher **Let's regroup/trade/exchange the 1 ten for 10 ones. See how 1 ten is the same as 10 ones?**

Students Yes.

Teacher **Now we have all these ones. But we can't leave the ones in the tens place. The tens place is only for tens. So, we place the 10 ones in the ones column. Where do we place the ones?**

Students In the ones column.

Teacher **Can we subtract __ ones now?**

Students Yes.

Teacher **Let's subtract __ ones.**

(Separate ones.)

Teacher **Now, let's subtract the tens of the subtrahend. How many tens do we need to subtract?**

Students __.

Teacher **Look at the tens of the minuend. Do we have enough tens in the minuend to subtract __ tens?**

Students Yes.

Teacher **We have enough tens. We do not have to regroup. Let's separate or subtract __ tens.**

(Separate tens.)

Teacher **So, let's count the remaining tens and ones to learn the difference. Ready?**

(Count the tens, then count the ones.)

Teacher **That means __ minus __ equals __. Let's say that together.**

Students __ minus __ equals __.

Teacher **Let's say it together again.**

Students __ minus __ equals __.

Teacher **So, if you have a set of __ and separate __ from the set, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher **What's a difference?**

Students The result of subtracting a subtrahend from a minuend.

Teacher **What does it mean to separate?**

Students To take away.

Teacher **How could you explain separating to a friend?**

Students We subtracted the ones but we didn't have enough ones so we regrouped 1 ten for 10 ones. Then, we subtracted the tens. We figured out the difference between __ and __.

Teacher **What's another way we could have solved this problem?**

Students We could have compared two sets.

2-DIGIT – 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher **Let's work on subtraction. What does it mean to subtract?**

Students To separate or compare.

Teacher **Subtraction means to separate from a set or compare two sets. Look at this problem.**

(Show problem.)

Teacher **First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**

Students To subtract.

Teacher **Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw a vertical line between the ones column and the tens column.**

(Draw vertical lines to separate place value columns.)

Teacher **Now, we start by subtracting. What should we subtract first?**

Students The ones.

Teacher **Which ones do we subtract?**

Students __ minus __.

Teacher **Do you have enough ones to subtract __ ones?**

Students No.

Teacher **We do not have enough ones. That means we have to regroup. To regroup, we take 1 ten and regroup/trade/exchange the 1 ten for 10 ones. To take 1 ten, I subtract 1 ten from the tens column. __ minus 1 equals __. I like to cross out the __ and write a __ in the tens column.**

(Show subtraction of 1 ten.)

Teacher **Now, I imagine regrouping this 1 ten into 10 ones. If I have 10 ones and add these ones to the __ ones, how many ones do I have now?**

Students __.

Teacher **I like to show the __ ones by crossing out the __ and writing __ in the ones column.**

(Show addition of 10 ones.)

Teacher **Now, let's subtract the ones. What's __ minus __?**

(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)

Students __.

Teacher **Yes! There are __ ones. Let's write __ below the equal line.**

(Write.)

Teacher **Now, let's subtract the tens. Which tens do we subtract?**

Students __ minus __.

Teacher **Do you have enough tens to subtract __ tens?**

Students Yes.

Teacher **You have enough tens to subtract or take away __ tens. We don't have to regroup. What's __ minus __?**

Students __.

Teacher **There are __ tens. Let's write __ below the equal line.**

(Write.)

Teacher **That means __ minus __ equals __. Let's say that together.**

Students __ minus __ equals __.

Teacher **Let's say it together again.**

Students __ minus __ equals __.

Teacher **So, if you have a set of __ and separate __ from the set, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**

Students The number from which another is subtracted.

Teacher **What's a subtrahend?**

Students The number to be subtracted.

Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the ones but we didn't have enough ones so we regrouped 1 ten for 10 ones. Then, we subtracted the tens. We figured out the difference between __ and __.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

Example

$$\begin{array}{r} 236 \\ -\ 89 \\ \hline 147 \end{array}$$

3-DIGIT – 2-DIGIT: ROUTINE WITHOUT MANIPULATIVES

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with our pencil. First, when I see a problem like this that requires computation, I like to draw vertical lines to separate the ones from the tens. Let's draw one vertical line between the ones column and the tens column and one vertical line between the tens column and the hundreds column.** (Draw vertical lines to separate place value columns.)
Teacher	**Now, we start by subtracting. What should we subtract first?**
Students	The ones.
Teacher	**Which ones do we subtract?**
Students	__ minus __.
Teacher	**Do you have enough ones to subtract __ ones?**
Students	No.
Teacher	**We do not have enough ones. That means we have to regroup. To regroup, we take 1 ten and regroup/trade/exchange the 1 ten for 10 ones. To take 1 ten, I subtract 1 ten from the tens column. 3 minus 1 equals 2. I like to cross out the 3 and write a 2 in the tens column.**

(Write 2 above tens column.)

Teacher **Now, I imagine regrouping this 1 ten into 10 ones. If I have 10 ones and add these ones to the 6 ones, how many ones do I have now?**

Students 16.

Teacher **I like to show the 16 ones by crossing out the 6 and writing 16 in the ones column.**

(Write 16 above ones column.)

Teacher **Now, let's subtract the ones. What's 16 minus 9?**

(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)

Students 7.

Teacher **Yes! 16 minus 9 equals 7. Let's write 7 below the equal line.**

(Write 7.)

Teacher **Now, let's subtract the tens. Which tens do we subtract?**

Students 2 minus 8.

Teacher **Do you have enough tens to subtract 8 tens?**

Students No.

Teacher **We do not have enough tens. That means we have to regroup. To regroup, we take 1 hundred and regroup/trade/exchange the 1 hundred for 10 tens. To take 1 hundred, I subtract 1 hundred from the hundreds column. 2 minus 1 equals 1. I like to cross out the 2 and write a 1 in the hundreds column.**

(Write 1 above hundreds column.)

Teacher **Now, I imagine regrouping this 1 hundred into 10 tens. If I have 10 tens and add these tens to the 2 tens, how many tens would you have?**

Students 12.

Teacher **It's helpful to show the 12 tens by crossing out the 2 and writing 12 in the tens column.**

(Write 12 above tens column.)

Teacher **Now, let's subtract the tens. What's 12 minus 8?**

(If a student has difficulty with subtraction, say: **Start with the subtrahend. Place that number in your fist, and let's count up to the minuend. Ready? __: __, __, __.** See Counting Up poster at the end of Module 7 for more information.)

Students 4.

Teacher **There are 4 tens. Let's write 4 below the equal line.**

(Write 4.)

Teacher **Are we finished subtracting?**

Students No.

Teacher **What do we subtract next?**

Students Hundreds.

Teacher **What do we subtract in the hundreds?**

Students 1 minus 0.

Teacher	**What's 1 minus 0?**
Students	1.
Teacher	**Let's write 1 below the equal line.** (Write 1.)
Teacher	**That means 236 minus 89 equals 147. Let's say that together.**
Students	236 minus 89 equals 147.
Teacher	**Let's say it together again.**
Students	236 minus 89 equals 147.
Teacher	**So, if you have a set of 236 and separate 89 from the set, the difference is 147. 236 minus 89 equals 147. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The amount between the minuend and subtrahend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How could you explain separating to a friend?**
Students	We subtracted the ones but we didn't have enough ones so we regrouped 1 ten for 10 ones. Then, we subtracted the tens but we didn't have enough tens so we regrouped 1 hundred for 10 ones. Then, we subtracted the hundreds. The difference between 236 and 89 is 147.
Teacher	**What's another way we could have solved this problem?**
Students	We could have compared two sets.

(3) Subtraction with Partial Differences* Algorithm

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
    - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative like a number line
    - Note that drawings can be used alongside or instead of manipulatives

*This algorithm requires an understanding of positive and negative numbers. If students have difficulty interpreting numbers less than 0, do not use this algorithm.

2-DIGIT – 2-DIGIT: ROUTINE

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's do this problem with our number line.** (Show number line.)
Teacher	**Our minuend is __. What's our minuend?**
Students	__.
Teacher	**We'll subtract the subtrahend from the minuend. What's our subtrahend?**
Students	__.
Teacher	**Let's subtract the subtrahend. In this example, we'll use the partial differences strategy. With partial differences, we subtract each place value and then combine the partial differences to find the difference.**
Teacher	**Let's first subtract the tens of the subtrahend. That means we have __ tens** (from the minuend) **minus __ tens** (from the subtrahend). **Think about this on the number line. What's __ minus __?**
Students	__.
Teacher	**__ is one of our partial differences. It's the difference of the tens. Let's write __ below the equal line. I like to write a positive/negative symbol because this number is positive/negative.** (Write.)
Teacher	**Now, let's subtract the ones of the subtrahend. How many ones do we subtract?**
Students	__.
Teacher	**Yes, let's subtract __ ones** (from the minuend) **minus __ tens** (from the subtrahend). **Think about this on the number line. What's __ minus __?**
Students	__.
Teacher	**__ is one of our partial differences. It's the difference of the ones. Let's write __ below the equal line. I like to write a positive/negative symbol because this number is positive/negative.** (Write.)
Teacher	**Now, below the equal line we have __ plus/minus __. What's __ plus/minus __?**
Students	__.
Teacher	**That means __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**Let's say it together again.**
Students	__ minus __ equals __.

Teacher	**So, if you have a set of __ and separate __, the difference is __. __ minus __ equals __. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How can you use the partial differences algorithm?**
Students	You subtract the tens for a partial difference. You subtract the ones for a partial difference. You then combine the partial differences to find the difference.

Example

$$\begin{array}{r} 236 \\ -\ 89 \\ \hline +200 \\ -50 \\ -3 \\ \hline 147 \end{array}$$

3-DIGIT – 2-DIGIT: EXAMPLE

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Let's use the partial differences algorithm. What's the partial differences strategy?**
Students	We find each partial difference in each place value column. Then, we combine the partial differences to find the difference.
Teacher	**What's our minuend?**
Students	236.
Teacher	**So, in this problem, we'll subtract the hundreds then tens then ones. How will we work on this problem?**
Students	Subtract the hundreds then tens then ones.
Teacher	**Let's start with the hundreds. How many hundreds do we subtract from 200?**

Students	0.
Teacher	**Yes! We have 0 hundreds to subtract. Let's write 200 under the equal line because we subtracted 0 from 200.**
	(Write 200.)
Teacher	**200 is one of our partial differences. What's 200?**
Students	The partial difference for the hundreds.
Teacher	**Let's subtract the tens of the subtrahend. How many tens do we need to subtract?**
Students	8 tens.
Teacher	**8 tens is the same as what?**
Students	80.
Teacher	**We subtract 80 from 30. What's 30 minus 80?**
Students	-50.
Teacher	**30 minus 80 is -50. Let's write -50 below the equal line.**
	(Write -50 below 200.)
Teacher	**-50 is one of our partial differences. It's the difference of the tens. What's -50?**
Students	The partial difference for the tens.
Teacher	**Now, let's subtract the ones of the subtrahend. How many ones do we need to subtract?**
Students	9 ones.
Teacher	**We subtract 9 ones from 6 ones. What's 6 minus 9?**
Students	-3.
Teacher	**6 minus 9 is -3. Let's write -3 below the equal line.**
	(Write -3 below -50.)
Teacher	**-3 is one of our partial differences. What's -3?**
Students	The partial difference for the ones.
Teacher	**Now, below the equal line we have 200 minus 50 minus 3. Let's do this in steps. What's 200 minus 50?**
Students	150.
Teacher	**What's 150 minus 3?**
Students	147.
Teacher	**Let's draw another equal line and write 147 below.**
	(Write 147.)
Teacher	**That means 236 minus 89 equals 147. Let's say that together.**
Students	236 minus 89 equals 147.
Teacher	**Let's say it together again.**
Students	236 minus 89 equals 147.
Teacher	**So, if you have a set of 236 and separate 89, the difference is 147. Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**

Students	The result of subtracting a subtrahend from a minuend.
Teacher	**What does it mean to separate?**
Students	To take away.
Teacher	**How can you use the partial differences algorithm?**
Students	You subtract the hundreds for a partial difference. Then, you subtract the tens for a partial difference. Then, you subtract the ones for a partial difference. You then combine to find the difference.

(4) Subtraction with Adding Up Algorithm

Routine

Materials:

- Module 8 Problem Sets
- Module 8 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- A hands-on tool or manipulative (e.g., money, Base-10 blocks)
  - Note that drawings can be used alongside or instead of manipulatives

2-DIGIT – 2-DIGIT: ROUTINE

Teacher	**Let's work on subtraction. What does it mean to subtract?**
Students	To separate or compare.
Teacher	**Subtraction means to separate from a set or to compare two sets. Look at this problem.** (Show problem.)
Teacher	**First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**
Students	To subtract.
Teacher	**Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?**
Students	The difference between two numbers.
Teacher	**So, in this problem, subtraction is the difference between what two numbers?**
Students	__ and __.
Teacher	**Let's figure out the difference between __ and __. Let's do this with our Base-10 blocks.** (Show Base-10 blocks.)
Teacher	**When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?**
Students	__.
Teacher	**Let's show the subtrahend with our Base-10 blocks. How many tens?**
Students	__.
Teacher	**How many ones?**

Students	__.
	(Show subtrahend with Base-10 blocks.)
Teacher	**Now, let's think about what we could add to the subtrahend to reach the minuend, __. I see that I could add __ ones to get to the nearest ten. I'll add the ones over here so I don't confuse this with the subtrahend ones.**
	(Add ones in separate pile.)
Teacher	**Now, what else could we add to reach the minuend, __? I see that I could add __ tens to get very close to the minuend of __. I'll add the tens over here so I don't confuse these tens with the subtrahend tens.**
	(Add tens.)
Teacher	**Have we reached the minuend yet?**
Students	No.
Teacher	**What could we add to reach the minuend?**
Students	__.
Teacher	**I could add __ ones to reach the minuend. Let's add the ones over here so I don't confuse these ones with the subtrahend ones.**
	(Add ones.)
Teacher	**So, the difference between __ and __ is: __, __, __, … What's the difference?**
Students	__.
Teacher	**That means __ minus __ equals __. Let's say that together.**
Students	__ minus __ equals __.
Teacher	**Let's say it together again.**
Students	__ minus __ equals __.
Teacher	**With this strategy, called adding up, you figure out the difference between __ and __ by adding up. You add up to find the difference between __ and __. How do you find the difference?**
Students	Adding up from __ to __.
Teacher	**Let's review. What's a minuend?**
Students	The number from which another is subtracted.
Teacher	**What's a subtrahend?**
Students	The number to be subtracted.
Teacher	**What's a difference?**
Students	The result of subtracting a subtrahend from a minuend.
Teacher	**How could you explain adding up to a friend?**
Students	You start with the subtrahend. You keep adding until you reach the minuend. You do this to find the difference between the minuend and subtrahend.

Example

236	89	
− 89	90	+1
	100	+10
	200	+100
	236	+36
		147

3-DIGIT – 2-DIGIT: EXAMPLE

Teacher **Let's work on subtraction. What does it mean to subtract?**

Students To separate or compare.

Teacher **Subtraction means to separate from a set or to compare two sets. Look at this problem.**

(Show problem.)

Teacher **First, I see a minus sign** (point). **The minus sign tells us to subtract. What does the minus sign mean?**

Students To subtract.

Teacher **Today, let's think about subtraction as the difference between two numbers. How can we interpret subtraction?**

Students The difference between two numbers.

Teacher **So, in this problem, subtraction is the difference between what two numbers?**

Students 236 and 89.

Teacher **Let's figure out the difference between 236 and 89.**

Teacher **When we think about subtraction as the difference between two numbers, let's start with our subtrahend. What's the subtrahend in this problem?**

Students 89.

Teacher **Let's write the subtrahend next to the problem. What should we write?**

Students 89.

Teacher **Now, let's think about what we could add to 89 to reach the minuend, 236. I see that I could add 1 one to get to the nearest ten. I'll write +1 over here to show I wanted to add 1.**

(Write +1.)

Teacher **If I added 1 to 89, what's the sum?**

Students 90.

Teacher **Let's write 90 below 89 to remember we're now at 90.**

(Write 90 below 89.)

Teacher **Let's figure out what we could add to 90 to reach the minuend, 236. Could we add 10 more to get to the nearest hundred?**

Students Yes.

Teacher **Let's write +10 to show we wanted to add 10.**

(Write +10 below +1.)

Teacher **If we added 10 to 90, what's the sum?**
Students 100.
Teacher **Let's write 100 below 90 to remember we're now at 110.**
(Write 100 below 90.)
Teacher **Let's keep going. What could we add to 100 to reach the minuend?**
Students 100.
Teacher **Great idea. Let's write +100 to show we wanted to add 100.**
(Write +100.)
Teacher **If I added 100 to 100, what's the sum?**
Students 200.
Teacher **Let's write 200 below 100 to remember we're now at 200.**
(Write 200 below 100.)
Teacher **Are we getting closer to 236?**
Students Yes.
Teacher **What could we add to 200 to reach the minuend, 236?**
Students 36.
Teacher **Let's write +36 to show we wanted to add 36.**
(Write +36.)
Teacher **If I added 36 to 200, what's the sum?**
Students 236.
Teacher **Let's write 236 below 200 to remember we're now at 236.**
(Write 236 below 200.)
Teacher **Did we reach the minuend?**
Students Yes!
Teacher **Now, we add +1 and +10 and +100 and +36 to determine the difference. How could we add these numbers?**
Students 100 + 36 + 10 + 1 (or other responses).
Teacher **So, the difference is 147. What's the difference?**
Students 147.
Teacher **That means 236 minus 89 equals 147. Let's say that together.**
Students 236 minus 89 equals 147.
Teacher **Let's say it together again.**
Students 236 minus 89 equals 147.
Teacher **With this strategy, called adding up, you figure out the difference between 236 and 89 by adding up. How do you find the difference?**
Students Adding up from 89 to 236.
Teacher **Let's review. What's a minuend?**
Students The number from which another is subtracted.
Teacher **What's a subtrahend?**
Students The number to be subtracted.
Teacher **What's a difference?**
Students The result of subtracting a subtrahend from a minuend.
Teacher **How could you explain adding up to a friend?**

Students	You start with the subtrahend. You keep adding until you reach the minuend. You do this to find the difference between the minuend and subtrahend.

D. Problems for Use During Instruction

See Module 8 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 8 Vocabulary Cards.

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Module 8:
Subtraction of Whole Numbers

Problem Sets

A. Two- and one-digit numbers without regrouping (5)
B. Two- and one-digit numbers with regrouping (5)
C. Two-digit numbers without regrouping (20)
D. Two-digit numbers with regrouping (20)
E. Three- and two-digit numbers without regrouping (5)
F. Three- and two-digit numbers with regrouping (5)
G. Three-digit numbers without regrouping (10)
H. Three-digit numbers with regrouping (10)

A.

$$\begin{array}{r} 68 \\ -\ \ 8 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 43 \\ -\quad 2 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 89 \\ -\quad 1 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 96 \\ -\ \ 5 \\ \hline \end{array}$$

A.

$$\begin{array}{r} 38 \\ -\ \ 7 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 61 \\ -\ \ 5 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 93 \\ -\ \ 6 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 45 \\ -\ \ 8 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 58 \\ -\ \ 9 \\ \hline \end{array}$$

B.

$$\begin{array}{r} 63 \\ -\quad 4 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 74 \\ -\ 31 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 84 \\ -\ 11 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 85 \\ -\ 70 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 97 \\ -\ 65 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 30 \\ -\ 20 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 91 \\ -\ 30 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 99 \\ -\ 38 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 55 \\ -\ 30 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 57 \\ -\ 10 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 98 \\ -\ 74 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 73 \\ -\ 32 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 85 \\ -\ 35 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 75 \\ -\ 62 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 77 \\ -\ 15 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 56 \\ -\ 26 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 65 \\ -\ 24 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 60 \\ -\ 30 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 97 \\ -\ 24 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 98 \\ -\ 40 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 69 \\ -\ 31 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 80 \\ -\ 24 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 72 \\ -\ 15 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 60 \\ -\ 58 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 75 \\ -\ 46 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 98 \\ -\ 79 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 96 \\ -\ 77 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 54 \\ -\ 46 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 80 \\ -\ 61 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 31 \\ -\ 18 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 71 \\ -\ 49 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 66 \\ -\ 59 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 26 \\ -\ 19 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 20 \\ -\ 16 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 96 \\ -\ 19 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 77 \\ -\ 18 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 75 \\ -\ 27 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 56 \\ -\ 49 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 78 \\ -\ 49 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 91 \\ -\ 47 \\ \hline \end{array}$$

D.

$$\begin{array}{r} 65 \\ -\ 57 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 195 \\ -\ 63 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 694 \\ -\ \ 20 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 384 \\ -\ \ 21 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 499 \\ -\ \ 18 \\ \hline \end{array}$$

E.

$$\begin{array}{r} 750 \\ -\ \ 20 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 172 \\ -\ 63 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 621 \\ -\ \ 12 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 735 \\ -\ \ 69 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 943 \\ -\ \ 51 \\ \hline \end{array}$$

F.

$$\begin{array}{r} 238 \\ -\ 54 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 747 \\ -\ 115 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 509 \\ -\ 301 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 773 \\ -\ 142 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 578 \\ -\ 427 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 685 \\ -\ 502 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 961 \\ -\ 151 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 323 \\ -\ 111 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 897 \\ -\ 530 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 888 \\ -\ 184 \\ \hline \end{array}$$

G.

$$\begin{array}{r} 350 \\ -\ 240 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 675 \\ -\ 328 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 582 \\ -\ 153 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 580 \\ -\ 321 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 777 \\ -\ 168 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 612 \\ -\ 223 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 202 \\ -\ 247 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 583 \\ -\ 108 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 490 \\ -\ 177 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 464 \\ -\ 215 \\ \hline \end{array}$$

H.

$$\begin{array}{r} 609 \\ -\ 134 \\ \hline \end{array}$$

Module 8:

Subtraction of Whole Numbers

Vocabulary Cards

algorithm
compare
computation
difference
equal sign
hundred column
minuend
minus sign
ones column
regroup/trade/exchange
separate
subtract/subtraction
subtrahend
tens column

algorithm

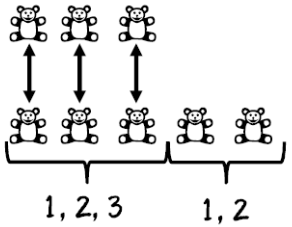
A procedure or description of steps that can be used to solve a problem.

---

compare

To find the difference between two sets.

5 – 3 = 2



computation

The action used to solve a problem.

---

difference

The result of subtracting one number from another number.

6 – 4 = 2

2 is the difference

equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

12 – 8 = 4

= is the equal sign

---

hundreds column

The column with digits in the hundreds place.

In the number 423, 4 is in the hundreds place.

minuend

The number from which another number is subtracted.

9 – 4 = 5

9 is the minuend

---

minus sign

The symbol that tells you to subtract.

9 – 4 = 5

– is the minus sign

ones column

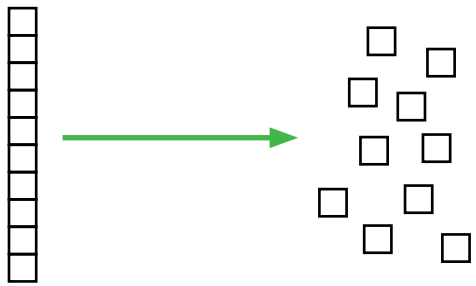
The column with digits in the ones place.

In the number 423, 3 is in the ones place.

---

regroup/trade/exchange

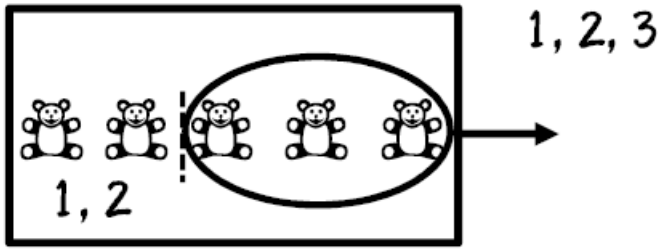
The process of exchanging 1 ten for 10 ones, 1 hundred for 10 tens, 1 thousand for 10 hundreds, etc.



separate

To start with a set and take away from that set.

5 – 3 = 2



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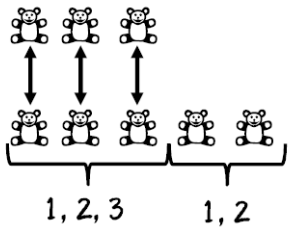
subtract/subtraction

To compare two sets or to separate from a set.

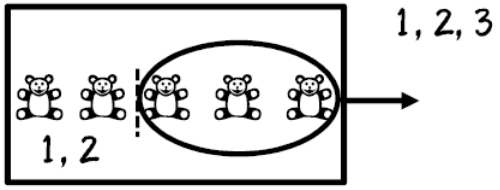
To compare two sets

5 – 3 = 2



To separate from a set

5 – 3 = 2



subtrahend

The number to be subtracted.

9 – 4 = 5

4 is the subtrahend

---

tens column

The column with digits in the tens place.

In the number 423, 2 is the in the tens column.