INCLUSION IN TEXAS

Instructional Routines for Mathematics Intervention

The purpose of these mathematics instructional routines is to provide educators with materials to use when providing intervention to students who experience difficulty with mathematics. The routines address content included in the grades 2-8 Texas Essential Knowledge and Skills (TEKS). There are 23 modules that include routines and examples – each focused on different mathematical content. Each of the 23 modules include vocabulary cards and problem sets to use during instruction. These materials are intended to be implemented explicitly with the aim of improving mathematics outcomes for students.

TEA Texas Education Agency

Instructional Routines for Mathematics Intervention

MODULE 10

Concepts of Multiplication

Module 10: Concepts of Multiplication

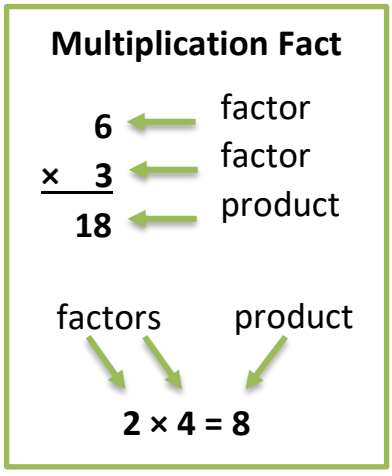
Mathematics Routines

A. Important Vocabulary with Definitions

Term	Definition
area	The number of square units that covers a closed figure.
array	A set of objects, pictures, or numbers arranged in columns and rows.
equal groups	Groups with the same number of objects or items in each group.
equal sign	The symbol that tells you that two sides of an equation are the same, balanced, or equal.
factor	A number that you multiply with another number to get the product.
multiply/multiplication	The process of adding a number to itself a number of times.
multiplication sign	The symbol that tells you to multiply.
partial products	The product of parts of each factor.
product	The result of multiplying two or more factors.

B. Background Information

Students need to learn two concepts of multiplication: (1) multiplication as equal groups and (2) multiplication as comparison. Typically, students first learn about multiplying as equal groups. Then, students learn about multiplying as comparison.



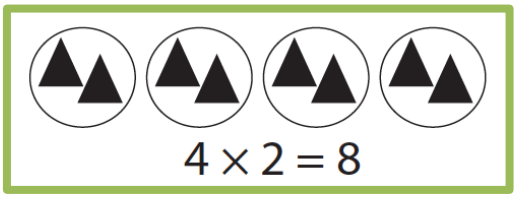
For learning the concepts of multiplication, we recommend using *mathematics facts*. We define a multiplication mathematics fact as single-digit factors multiplied for a single- or double-digit product. You may present multiplication facts vertically or horizontally.

C. Routines and Examples

(1) Multiplication as Equal Groups

Routine

Materials:



- Module 10 Multiplication Problems
- Module 10 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- Any hands-on tool or manipulative (e.g., cubes, clips) and any container (e.g., plates, cups)

Teacher	**Let's work on multiplication. Today, let's think about multiplication as equal groups. What does equal mean?**
Students	The same.
Teacher	**What is a group?**
Students	A collection of items.
Teacher	**So, with equal groups, we'll have the same number in each group. What's an equal group?**
Students	Same number in each group.
Teacher	**When we create equal groups, we'll put the same number in each group. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**We'll multiply by creating equal groups. In a multiplication problem, we'll use the first factor to tell us the number of groups. What will the first factor tell us?**
Students	The number of groups.
Teacher	**And we'll use the second factor to tell us how many in each group. What will the second factor tell us?**
Students	How many in each group.
Teacher	**Great. Let's do this problem.** (Move clips to workspace.)
Teacher	**Our first factor is __. What's our first factor?**
Students	__.
Teacher	**Let's show this factor by showing __ groups. We'll show the groups with the plates.** (Use plates to show groups.)
Teacher	**How many groups?**

Students __.

Teacher **Our second factor is __. What's our second factor?**

Students __.

Teacher **Let's show the second factor by placing __ clips in each group. Remember we need to show the same, or equal, number of clips in each group.**

(Show clips in groups.)

Teacher **How many clips in each group?**

Students __.

Teacher **So, we have __ groups of __. Let's multiply by counting all the clips. How could we count the clips?**

Students Count all the clips or count by groups.

Teacher **Yes. Let's count by groups to learn the product. We have __ groups of __ so that's __, __, __, …**

(Count clips by groups – skip count.)

Teacher **How many clips are there altogether?**

Students __.

Teacher **Yes! There are __ clips. So, __ times __ equals __. Let's say that together.**

Students __ times __ equals __.

Teacher **Let's say it together again.**

Students __ times __ equals __.

Teacher **So, if you have __ groups with an equal number of __ in each group, the product is __. __ times __ equals __. Let's review. What's a factor?**

Students One of the numbers multiplied in a multiplication problem.

Teacher **What's a product?**

Students The result of multiplying factors.

Teacher **What does it mean to make equal groups?**

Students To have groups with an equal number in each group.

Teacher **How could you explain multiplying to a friend?**

Students We started groups and placed the same number of clips in each group. The product was the total number of clips.

Example

$$\begin{array}{r} 4 \\ \times\ 3 \\ \hline 12 \end{array}$$

Teacher **Let's work on multiplication. Today, let's think about multiplication with equal groups. What does it mean to make equal groups?**

Students Show groups with an equal number in each group.

Teacher **When we make equal groups, we show groups with an equal number in each group. Look at this problem.**

(Show problem.)

Teacher **First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**

Students To multiply.

Teacher **We'll multiply by making equal groups. Let's show the first factor with our groups or plates. How will we show the first factor?**

Students As the groups with our plates.

Teacher **Our first factor is 4. What's our first factor?**

Students 4.

Teacher **Let's show this factor by showing 4 groups.**

(Show 4 groups by showing 4 plates.)

Teacher **How many groups?**

Students 4.

Teacher **Our second factor is 3. What's our second factor?**

Students 3.

Teacher **Let's show the second factor by 3 placing cubes in each group.**

(Place 3 cubes in each group.)

Teacher **How many cubes in each group?**

Students 3.

Teacher **So, we have 4 groups of 3 or 4 times 3. Let's multiply to learn the product. Let's count the cubes. How could we count?**

Students We could count all of the cubes.

Teacher **We could count all the cubes. Let's do that together.**

Students 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Teacher **We could also skip count the cubes in groups. Let's do that together.**

Students 3, 6, 9, 12.

Teacher **What's the product?**

Students 12.

Teacher **How many cubes are there altogether?**

Students 12.

Teacher **Yes! There are 12 cubes. So, 4 groups of 3 equals 12. Let's say that together.**

Students 4 groups of 3 equals 12.

Teacher **We could also say 4 times 3 equals 12. Let's say that together.**

Students 4 times 3 equals 12.

Teacher **Let's say it together again.**

Students 4 times 3 equals 12.

Teacher **So, if you have 4 groups with 3 in each group, the product is 12. 4 times 3 equals 12. Let's review. What's a factor?**

Students One of the numbers multiplied in a multiplication problem.

Teacher **What's a product?**

Students The total when you multiply groups with an equal number in each group.

Teacher **What does it mean to make equal groups?**

Students To have groups with an equal number in each group.

Teacher **How could you explain multiplying to a friend?**

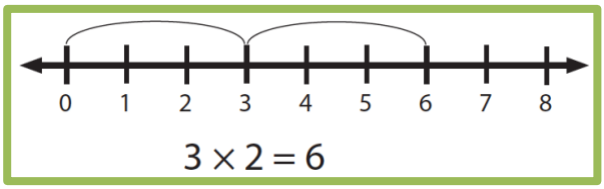
Students We started groups and placed the same number of cubes in each group. The product was the total number of cubes.

(2) Multiplication as Comparison

Routine

Materials:

- Module 10 Problems
- Module 10 Vocabulary Cards
  - If necessary, review Vocabulary Cards before teaching
- Number line



Teacher	**Let's work on multiplication. Today, let's think about multiplication as comparison. What does comparison mean?**
Students	To look at one set compared to another set.
Teacher	**In subtraction, we compare two sets by determining the difference between two numbers. In multiplication, we compare two sets by multiplying a first set and a second set. How do we compare in multiplication?**
Students	Multiply two sets together.
Teacher	**When we multiply by comparison, we have a set. Imagine you have a set of 5 apples. Your friend has 4 times as many apples as you. We multiply to figure out how many apples your friend has. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Today we'll multiply by comparison, but there are other ways to multiply – like using equal groups. Let's start by getting out our number line.** (Move number line to workspace.)
Teacher	**Our first factor is __. What's our first factor?**
Students	__.
Teacher	**The first factor is our set. What's the first factor?**
Students	Our set.
Teacher	**Our second factor is __. What's our second factor?**
Students	__.
Teacher	**The second factor tells us how many times to multiply the first factor. What does the second factor tell us?**
Students	How many times to multiply the set.
Teacher	**So, let's show the first set on the number line. We have a set of __ so let's count out __.** (Count first set on number line.)
Teacher	**Now, let's multiply that set __ times. How many times?**
Students	__.

Teacher	**To multiply, let's count the original set __ times. Watch me: __, __, __, …** (Count sets by multiplying.)
Teacher	**The product is the last number we said. We counted to __. What's the product?**
Students	__.
Teacher	**How many altogether?**
Students	__.
Teacher	**Yes! There are __. So, __ times __ equals __. Let's say that together.**
Students	__ times __ equals __.
Teacher	**Let's say it together again.**
Students	__ times __ equals __.
Teacher	**So, if you have a set of __ and multiply that set __ times, the product is __. __ times __ equals __. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The total when you multiply groups with an equal number in each group.
Teacher	**What does it mean to multiply by comparison?**
Students	To have a set and multiply the set a number of times.
Teacher	**How could you explain multiplying to a friend?**
Students	We started a set and counted the set a number of times on the number line. The product was the total.

Example

$$\begin{array}{r} 4 \\ \times\ 3 \\ \hline 12 \end{array}$$

Teacher	**Let's work on multiplication. Today, let's think about multiplication by comparison. What does it mean to compare?**
Students	To have a set and compare that set a number of times.
Teacher	**When we compare, we start with a set and multiply that set a number of times. Look at this problem.** (Show problem.)
Teacher	**First, I see a multiplication sign** (point). **The multiplication sign tells us to multiply. What does the multiplication sign mean?**
Students	To multiply.
Teacher	**Today we'll multiply by comparison, but there are other ways to multiply – like with equal groups. Let's start by getting out our number line. Let's do this together.** (Move number line to workspace.)
Teacher	**Our first factor is 4. What's our first factor?**

Students	4.
Teacher	**Our second factor is 3. What's our second factor?**
Students	3.
Teacher	**That means we're going to multiply the set of four 3 times. What does our problem mean?**
Students	Multiply the set of 4 3 times.
Teacher	**Ready? Let's use the number line to count the set of four 3 times. I show one set of 4** (place finger on 4), **a second set of 4** (place finger on 8), **and a third set of 4** (place finger on 12). **What's the last number we said?**
Students	12.
Teacher	**The product is the last number we said. We counted 12. What's the product?**
Students	12.
Teacher	**So, 4 times 3 equals 12. Let's say that together.**
Students	4 times 3 equals 12.
Teacher	**Let's say it together again.**
Students	4 times 3 equals 12.
Teacher	**So, if you have a set of 4 and multiply that set 3 times, the product is 12. 4 times 3 equals 12. Let's review. What's a factor?**
Students	One of the numbers multiplied in a multiplication problem.
Teacher	**What's a product?**
Students	The total when you multiply groups with an equal number in each group.
Teacher	**What does it mean to multiply by comparison?**
Students	To have a set and multiply the set a number of times.
Teacher	**How could you explain multiplying to a friend?**
Students	We started 4 and multiplied four 3 times. 4 times 3 equals 12.

D. Problems for Use During Instruction

See Module 10 Problem Sets.

E. Vocabulary Cards for Use During Instruction

See Module 10 Vocabulary Cards.

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Module 10:

Concepts of Multiplication

Problem Sets

A. Single-digit multiplication facts (60)

$$\begin{array}{r} 1 \\ \times\ 0 \\ \hline \end{array}$$

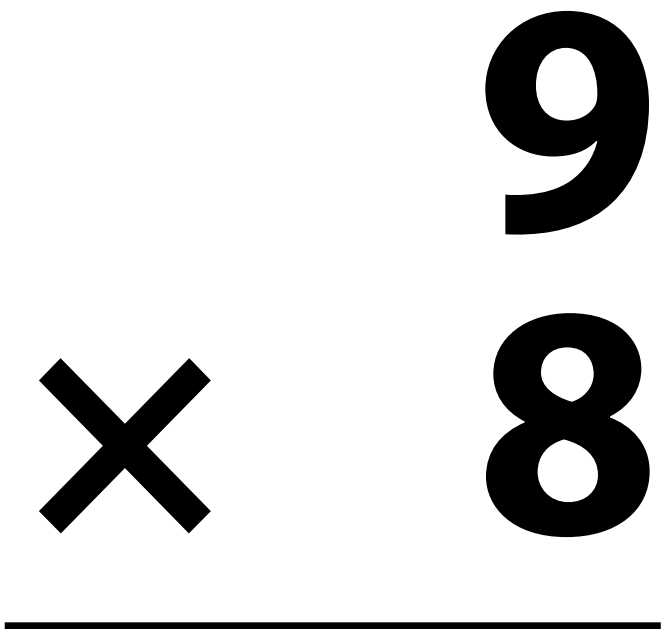
$$\begin{array}{r} 5 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times\ 6 \\ \hline \end{array}$$

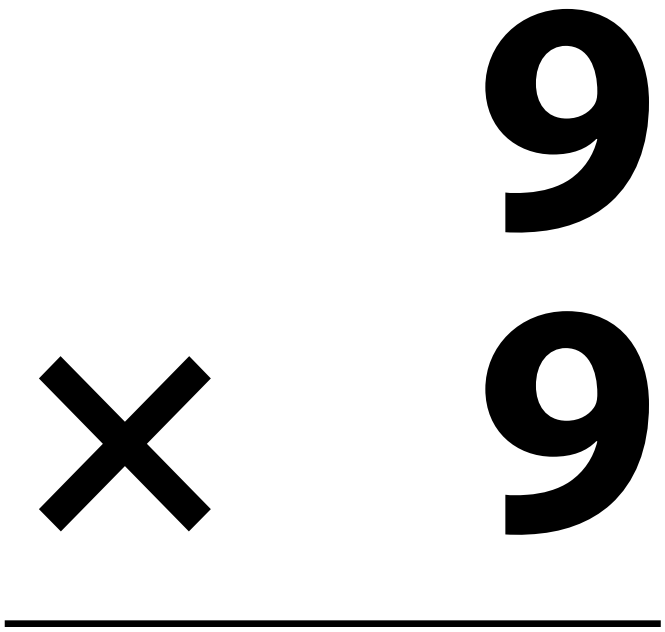
$$\begin{array}{r} 4 \\ \times\ 2 \\ \hline \end{array}$$



$$\begin{array}{r} 3 \\ \times\ 3 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times\ 5 \\ \hline \end{array}$$


9
× 9

$$\begin{array}{r} 2 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times\ 8 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 4 \\ \times \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times\ 6 \\ \hline \end{array}$$

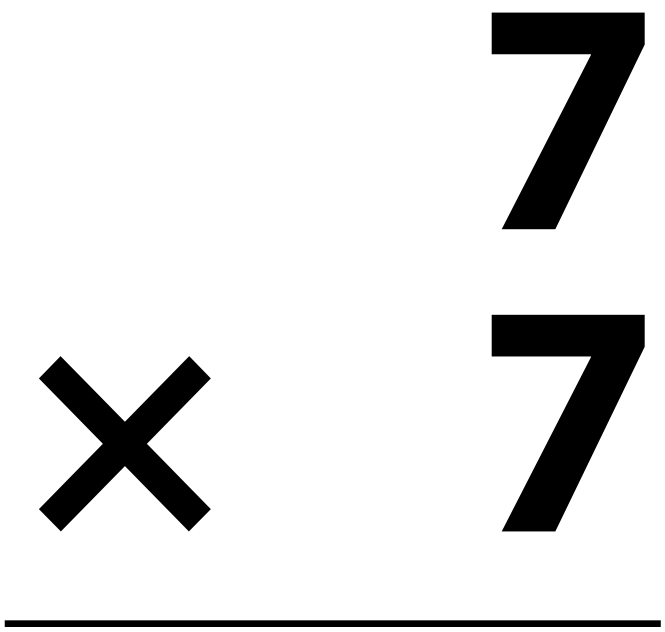
$$\begin{array}{r} 8 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times\ 0 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times\ 7 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times \quad 3 \\ \hline \end{array}$$


7
× 7

$$\begin{array}{r} 0 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times\ 0 \\ \hline \end{array}$$

$$\begin{array}{r} 7 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 9 \\ \times\ 7 \\ \hline \end{array}$$

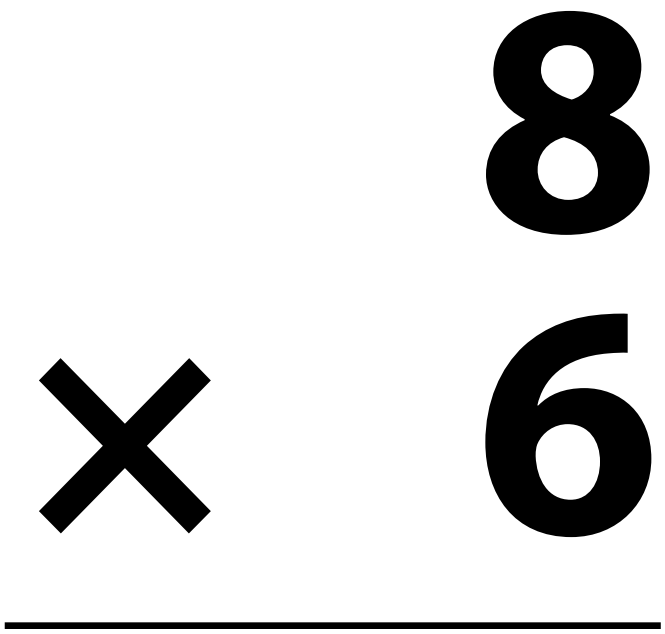
$$\begin{array}{r} 4 \\ \times\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ \times \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times \quad 1 \\ \hline \end{array}$$

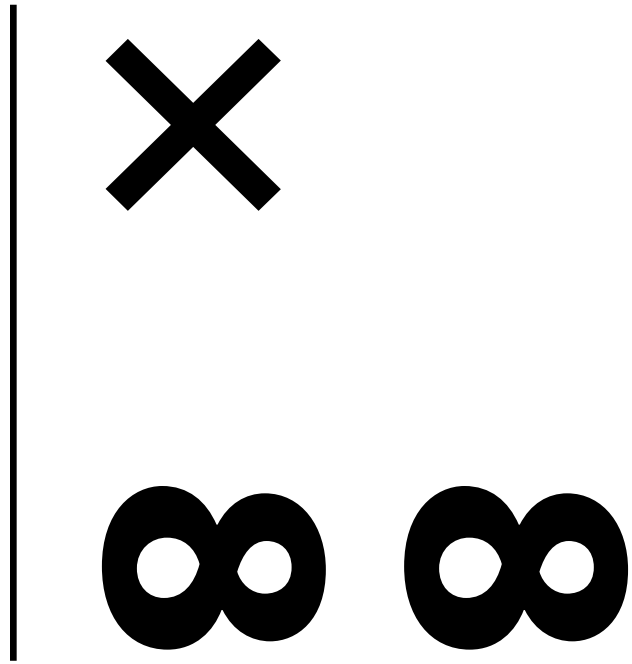
$$\begin{array}{r} 6 \\ \times \quad 5 \\ \hline \end{array}$$


8
× 6

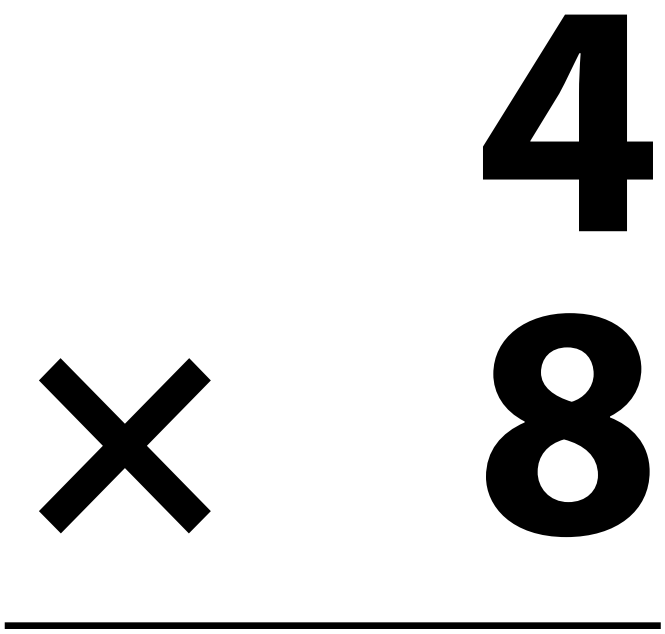
$$\begin{array}{r} 0 \\ \times \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times\ 3 \\ \hline \end{array}$$



$$\begin{array}{r} 5 \\ \times\ 5 \\ \hline \end{array}$$


4
× 8

$$\begin{array}{r} 5 \\ \times\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ \times \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times \quad 9 \\ \hline \end{array}$$

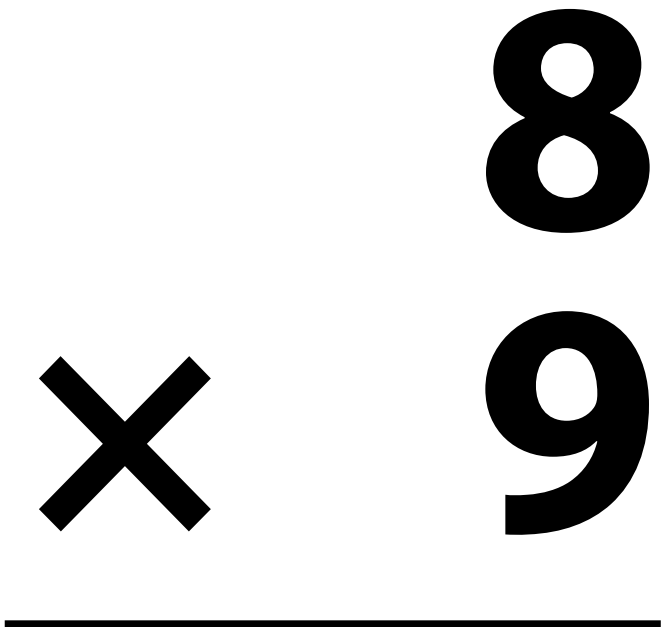
$$\begin{array}{r} 6 \\ \times \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ \times \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \times\ 7 \\ \hline \end{array}$$

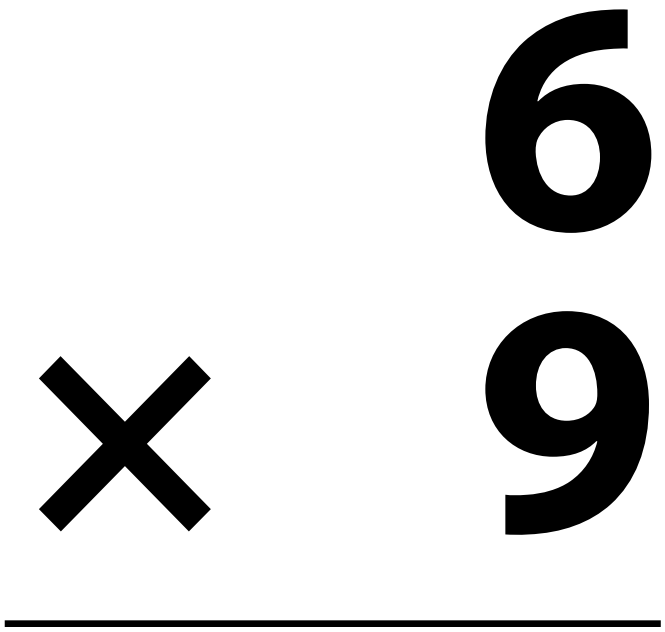
$$\begin{array}{r} 2 \\ \times\ 2 \\ \hline \end{array}$$

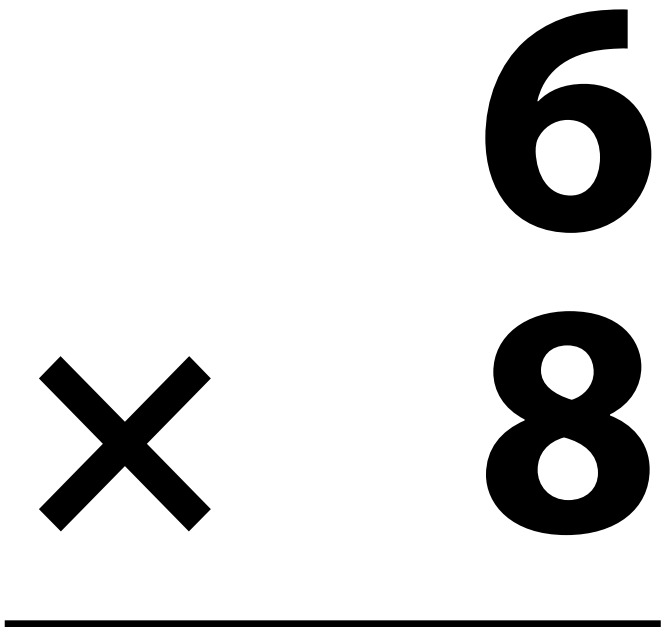


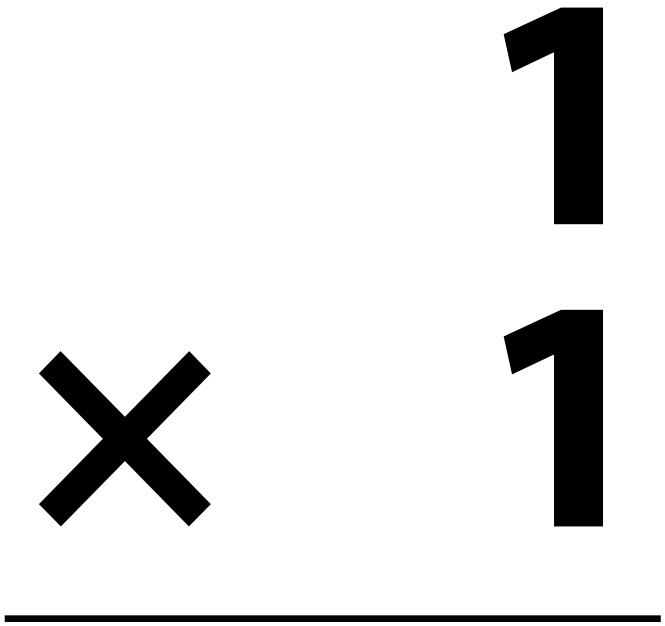
$$\begin{array}{r} 1 \\ \times\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ \times \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 0 \\ \times\ 0 \\ \hline \end{array}$$







$$\begin{array}{r} 5 \\ \times\ 6 \\ \hline \end{array}$$

Module 10:

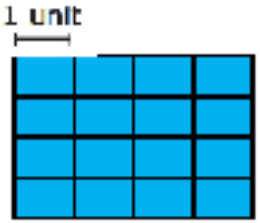
Concepts of Multiplication

Vocabulary Cards

area
array
equal groups
equal sign
factor
multiply/multiplication
multiplication sign
partial products
product

area

The number of square units that covers a closed figure.



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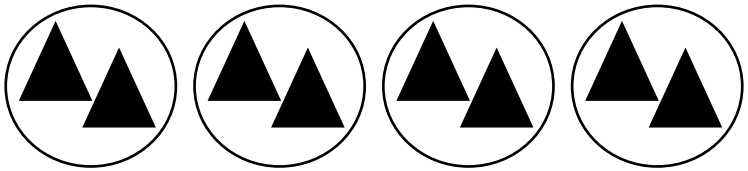
array

A set of objects, pictures, or numbers arranged in columns and rows.

equal groups

Groups with the same number of objects or items in each group.



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equal sign

The symbol that tells you that two sides of an equation are the same, balanced, or equal.

$$2 \times 8 = 16$$

= is the equal sign

factor

A number you multiply with another number to get the product.

$2 \times 8 = 16$

2 and 8 are the factors

---

multiply/multiplication

The process of adding a number to itself a number of times.

$4 \times 2 = 8$

multiplication sign

The symbol that tells you to multiply.

$$2 \times 8 = 16$$

$\times$ is the multiplication sign

---

partial products

The product of parts of each factor.

$$\begin{array}{r} 13 \\ \times\ 45 \\ \hline 400 \quad (40 \times 10) \\ 120 \quad (40 \times 3) \\ 50 \quad (10 \times 5) \\ +\ 15 \quad (5 \times 3) \\ \hline 585 \end{array}$$

product

The result of multiplying two or more factors.

$$2 \times 8 = 16$$

16 is the product